Tanzania Journal of Science 44(3): 103-114, 2018 ISSN 0856-1761, e-ISSN 2507-7961

EFFECTS OF MAGNETIC FIELDS ON AN UNSTEADY MIXED CONVECTIVE BOUNDARY LAYER FLOW OF AN ELECTRICALLY CONDUCTING FLUID WITH TEMPERATURE DEPENDENT PROPERTIES

Raymond Kitengeso[1], Eunice Mureithi[2], Makungu James[2], and John Mango[3]
[1]Department of Mathematics, University of Dodoma, Tanzania
[2]Department of Mathematics, University of Dar es Salaam, Tanzania
[3]Department of Mathematics, Makerere University, Uganda

ABSTRACT

A mixed convective boundary layer flow of an electrically conducting fluid with temperature dependent properties over an inclined plate is investigated. The magnetohydrodynamic boundary layer governing equations are derived by using Boussinesq and boundary layer approximations. The equations are transformed to similarity form using a similarity transformation variable and the resulting boundary value problem is solved numerically. The effects of magnetic fields, unsteadiness, mixed convection and variable fluid properties on velocity and temperature in the boundary layer are analysed. The effects of pertinent parameters on skin friction and heat transfer are also analysed.

Keywords: Magnetohydrodynamics; Mixed convection; Boundary layer; Temperature dependent fluid properties.

INTRODUCTION

Investigation of the effects of magnetic fields on an unsteady mixed convection boundary layer flow has attracted the attention of many researchers in recent years. This is due to the significance of such effects in engineering, transportation, medicine, geophysics and in academics.

Boundary layer refers to the thin layer of fluid formed on the flow surface in which the velocity of the fluid increases from zero at the surface (no slip condition) to its full value which corresponds to external frictionless flow. The two types of boundary layers to be considered are the velocity boundary layer and thermal boundary layer. Velocity boundary layer may be expected to occur in conjunction with thermal boundary layer. The theory boundary layers was developed by Ludwig Prandtl in 1904 (Schlichting 1979). The thickness of a boundary layer depends on various factors. These include fluid properties such as viscosity, velocity, temperature, steadiness of the flow as well as the nature, orientation, and motion of the surface.

The effects of internal and external body forces on the boundary layer flow lead to natural and forced convection, respectively. Forced convection is heat transfer driven by external body forces such as a pump or a fan, while natural convection is driven by internal body forces due to heating or cooling exerted directly within the fluid. The situation where the ratio of the strength of natural convection flow to the strength of forced convection flow is of order one is referred to as a mixed convection flow (Lienhard IV and Lienhard V 2015). Various studies on mixed convection flow have been done by researchers such as Roy and Singh (2007), Mureithi (2014) and Das et al. (2015). The results indicated that buoyancy forces affect the velocity within the boundary layer.

The study of mutual interaction between magnetic fields and flow of electrically conducting fluids such as liquid metals, ionized gas and salt water (strong electrolytes) is commonly referred to as magnetohydrodynamics (MHD). The combination of electromagnetic and hydrodynamic principles for continuous medium forms the governing equations for magnetohydrodynamics (Schnack 2009). The study of magnetic fields effects on natural and man-made flows goes back to the discovery of Alfven waves, plasma physics, and engineering applications (Davidson 2001). Investigation of the effects of magnetic fields on boundary layer flow has obtained similar results in various analysis, that the presence of magnetic fields causes deceleration of the fluid motion and decreases the transfer of heat (Kafoussias and Nanousis 1997, Raju et al. 2015)

Unsteady boundary layer occurs as a result of different scenarios of the fluid flow, such as a body moving through the fluid at rest; the external fluid varying with time and moving past the body at rest; the fluid being at rest and the body executing periodic motion, or the body being at rest and the fluid performing periodic motion (Schlichting 1979). Various studies on unsteady boundary layer flow have been carried out by researchers such as Nazar et al. 2004, Roy and Singh 2007 and Vajravelu et al. 2013. They found that increasing the unsteady parameter leads to a decrease in the thickness of velocity boundary layer and the thermal boundary layer and there is a smooth transition from steady to unsteady state.

Various studies on the effects of magnetic fields on an unsteady mixed convective boundary layer flow have also been done by researchers such as Das et al. (2015), Hua and Su (2015) and Reddy (2016). Their results show that the presence of magnetic fields has significant effect on the unsteady mixed convective boundary layer flow. Das et al. (2015) studied the effects of magnetic fields on mixed convective slip flow over an inclined plane with combined effects of ohmic heating and viscous dissipation which affect velocity within the boundary layer. The study of Hua and Su (2015) analyzed the effects of magnetic field on unsteady boundary layer flow aiming at formulating and proving theorems on the analytical properties of dimensionless velocity. Recently Reddy (2016) examined the effects of magnetic fields on unsteady boundary layer flow over an infinite porous plate. Available studies do not examine the effects of temperature dependent fluid properties. This study extends the work done by Das et al. (2015) by incorporating the effects of temperature dependent fluid properties on an unsteady mixed convective boundary layer flow of an electrically conducting fluid with temperature dependent properties.

MATERIALS AND METHODS

Consider an unsteady, two-dimensional flow of a viscous incompressible fluid over a heated inclined plate under the influence of transverse magnetic field, $\boldsymbol{B_0}$. The x-axis is taken along the inclined plate and the y-axis is normal to the x-axis. It is assumed that initially the fluid is at rest and that viscosity and thermal conductivity are temperature dependent.

A schematic diagram for the flow is shown below.

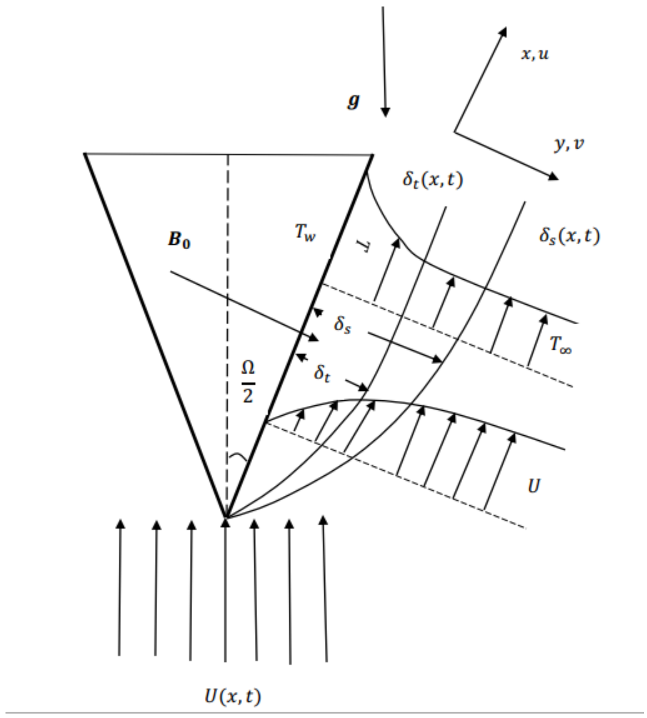


Figure 1: Velocity and thermal boundary layer on an inclined plane

We define the velocity vector $\boldsymbol{u} = (u(x,y,t), v(x,y,t))$. Here Ω is the inclination angle, δ_s is the momentum boundary layer thickness, δ_t is the thermal boundary layer thickness, $\Delta T = T_w - T_\infty$, $U(x,t)$ is the free stream velocity, T_∞ is the free stream temperature, $T(x,y,t)$ is the temperature inside thermal boundary layer and the temperature of the plate T_w is assumed to be constant.

Under the above conditions and assumptions the boundary layer equations in two dimensions with variable viscosity and thermal conductivity becomes:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \rho\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x}\right) + \rho g\beta(T - T_\infty)\cos\frac{\Omega}{2} + \frac{\partial}{\partial y}\left(\mu(T)\frac{\partial u}{\partial y}\right) - \sigma B_0^2(u - U), \tag{2}$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k(T)\frac{\partial T}{\partial y}\right) + \mu(T)\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_0^2(u - U)^2. \tag{3}$$

The initial conditions $(t \leq 0)$ are given by:

$$u(x,y,0) = 0, \qquad v(x,y,0) = 0, \qquad T(x,y,0) = T_\infty. \tag{4}$$

The boundary conditions $(t > 0)$ are given by:

$$\begin{aligned} &u(x,0,t) = 0,\ v(x,0,t) = 0,\ T(x,0,t) = T_w \text{ at } y = 0, \\ &u(x,\infty,t) = 0,\ v(x,\infty,t) = 0,\ T(x,\infty,t) = T_\infty \text{ at } y \to \infty. \end{aligned} \tag{5}$$

Here g is the gravitational acceleration, β is the coefficient of thermal expansion, c_p is the specific heat at constant pressure and ρ is the fluid density at free stream temperature T_∞.

Viscosity $\mu(T)$ has been assumed to vary exponentially with temperature (Mureithi 2014) as in most common fluids. So we use Arrhenius model which takes the exponential form:

$$\mu(T) = \mu_\infty e^{-\gamma(T-T_\infty)}. \tag{6}$$

where; μ_∞ and μ_w are fluid viscosity at the free stream temperature, T_∞ and at the wall temperature T_w respectively. γ is the viscosity variation parameter, for gases $\gamma < 0$ and liquids $\gamma > 0$

$$\gamma = \frac{1}{T_w - T_\infty} \ln\left(\frac{\mu_\infty}{\mu_w}\right) \qquad \mu_w < \mu_\infty.$$

$\alpha = \gamma(T_w - T_\infty) = \gamma\Delta T = \ln\left(\frac{\mu_\infty}{\mu_w}\right)$ is a very small parameter.

Thermal conductivity $k(T)$ has been assumed to vary linearly with temperature (Vajravelu et al. 2013) and is given by

$$k(T) = k_\infty\left(1 + \epsilon\left(\frac{T - T_\infty}{T_w - T_\infty}\right)\right) \tag{7}$$

where; ϵ is a small parameter known as the variable thermal conductivity parameter and k_∞ is the free stream thermal conductivity.

Boundary layer similarity equations

The continuity equation (1) is satisfied by introducing a stream function $\psi(x,y,t)$ through the:

$$u = \frac{\partial\psi}{\partial y}, \qquad v = -\frac{\partial\psi}{\partial x} \tag{8}$$

To transform the governing equations into similarity form, we define the following scaling transformations:

$$\psi = U(x,t)R(x,t)f(\eta), \qquad T(x,y,t) = T_\infty + (T_w - T_\infty)\theta(\eta), \qquad \eta = \frac{y}{R(x,t)}$$

where η is the boundary layer similarity variable, y-coordinate is referenced to the non-dimensional scale $R(x,t)$ since it is related to the boundary layer growth.

The system of equations (1-3) is reduced to similarity form, it is obtained that:

$$U(x,t) = C\left(\frac{x}{t}\right)^m, R(x,t) = \sqrt{\left(\frac{2}{m+1}\right)\frac{\upsilon x}{U(x,t)}}\ , \ \psi(x,y,t) = f(\eta)\sqrt{\left(\frac{2}{m+1}\right)\upsilon x U(x,t)}$$

and $(x,y,t) = y\sqrt{\left(\frac{m+1}{2}\right)\frac{U(x,t)}{\upsilon x}}$.

The angle of the inclined plate $\Omega = \pi B$ is related to m under the expression given by $m = \frac{\frac{\Omega}{2}}{\pi - \frac{\Omega}{2}}$ which implies that, if $m = 0, m = \frac{1}{3}, m = \frac{1}{2}$ and $m = 1$ corresponds to flow over a plate at an angle of $\Omega = 0$ (vertical plate), $\Omega = \frac{\pi}{2}, \Omega = \frac{2\pi}{3}$ and $\Omega = \pi$ (stagnation flow at a horizontal plate) respectively.

The similarity equations take the form:

$$f''' = \alpha\theta' f'' - \left(\frac{2}{m+1}\right)e^{\alpha\theta}\theta\lambda\cos\frac{\Omega}{2} + \left(\frac{1}{m+1}\right)St\, e^{\alpha\theta}(1 - f' - \eta f'') + \left(\frac{2m}{m+1}\right)e^{\alpha\theta}((f')^2 - 1) - e^{\alpha\theta}ff'' + \left(\frac{2}{m+1}\right)M\, e^{\alpha\theta}(f' - 1), \tag{9}$$

$$\theta'' = -StPr\left(\frac{1}{m+1}\right)\left(\frac{\eta\theta'}{1+\epsilon\theta}\right) - Pr\left(\frac{\theta' f}{1+\epsilon\theta}\right) - \left(\frac{\epsilon(\theta')^2}{1+\epsilon\theta}\right) - PrEc\left(\frac{e^{-\alpha\theta}(f'')^2}{1+\epsilon\theta}\right) - MPrEc\left(\frac{2}{m+1}\right)\left(\frac{(f'-1)^2}{1+\epsilon\theta}\right). \tag{10}$$

Subject to the boundary conditions:

$$f'(\eta = 0) = 0,\ f(\eta = 0) = 0,\ \theta(\eta = 0) = 1$$
$$f'(\eta \to \infty) = 1,\ \theta(\eta \to \infty) = 0. \tag{11}$$

where: M and λ are the local magnetic field parameter and the mixed convection parameter, respectively, defined by

$$M = \frac{x\sigma B_0^2}{U\rho},\ \lambda = \frac{Gr_x}{(Re_x)^2}$$

Gr_x is the local Grashof number and Re_x is the local Reynolds number.

St is the unsteady parameter, Ec is the Eckert number, and Pr is the Prandtl number.

The shear stress and the heat transfer at the inclined plane can be represented using the local skin friction coefficient and the local Nusselt number, defined by

$$C_f(Re_x)^{1/2} = e^{-\alpha}f''(0)\sqrt{2(m+1)}$$

and

$$Nu_x(Re_x)^{-1/2} = -(1+\epsilon)\theta'(0)\sqrt{\left(\frac{m+1}{2}\right)}$$

Numerical solution of the boundary layer similarity equations

The system of nonlinear first order ordinary differential equations (9-10) with boundary conditions (11) are solved numerically using *bvp4c* with MATLAB package. This method was also been used by Raju et al. (2015). The *bvp4c* is a finite difference code that implements the three-stage Lobatto III formula. This is a collocation formula and the collocation polynomial provides a C^1-continuous solution that is fourth-order accurate uniformly in $[a, b]$. Mesh selection and error control are based on the residual of the continuous solution (Shampine et al. 2003).

The non-linear system is reduced to a system of first order ordinary differential equations by setting:

$f_1 = f,\quad f_2 = f',\quad f_3 = f'',\quad f_4 = \theta$ and $f_5 = \theta'$
$f_1' = f' = f_2,\quad f_2' = f'' = f_3,\quad f_3' = f''',\quad f_4' = \theta' = f_5$ and $f_5' = \theta''$
The system of first order ordinary differential equations then becomes:

$$f_1' = f_2, \tag{12}$$

$$f_2' = f_3, \tag{13}$$

$$f_3' = \alpha f_5 f_3 - \left(\frac{2}{m+1}\right) e^{\alpha f_4} f_4 \lambda \cos\frac{\Omega}{2} + \left(\frac{1}{m+1}\right) St\, e^{\alpha f_4}(1 - f_2 - \eta f_3) + \left(\frac{2m}{m+1}\right) e^{\alpha f_4}(f_2^2 - 1) - e^{\alpha f_4} f_1 f_3 + \left(\frac{2}{m+1}\right) M e^{\alpha f_4}(f_2 - 1), \tag{14}$$

$$f_4' = f_5, \tag{15}$$

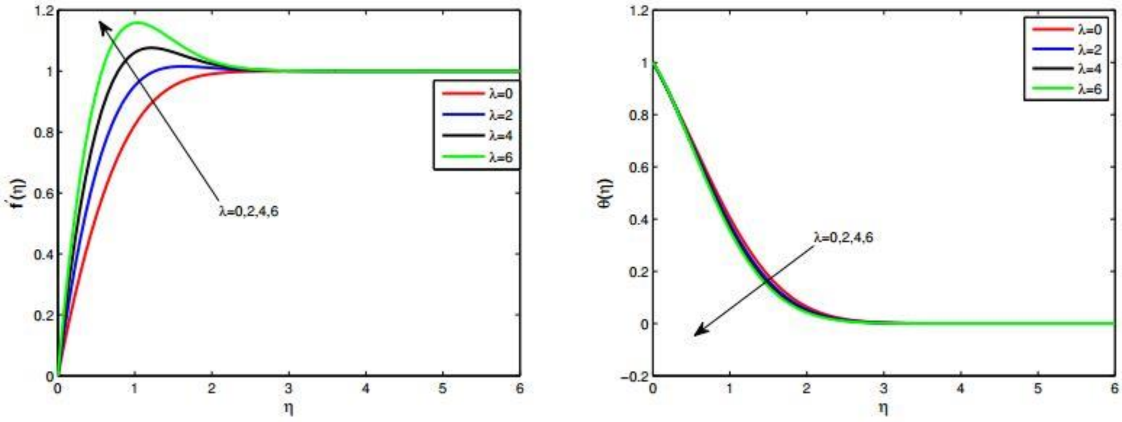
$$f_5' = -StPr\left(\frac{1}{m+1}\right)\left(\frac{\eta f_5}{1+\epsilon f_4}\right) - Pr\left(\frac{f_5 f_1}{1+\epsilon f_4}\right) - \left(\frac{\epsilon f_5^2}{1+\epsilon f_4}\right) - PrEc\left(\frac{e^{-\alpha f_4} f_3^2}{1+\epsilon f_4}\right) - MPrEc\left(\frac{2}{m+1}\right)\left(\frac{(f_2-1)^2}{1+\epsilon f_4}\right). \tag{16}$$

Subject to the boundary conditions:

$$\text{at } \eta = 0 \qquad f_1' = f_2 = 0,\quad f_1 = 0,\quad f_4 = 1\,. \tag{17}$$
$$\text{as } \eta \to \infty \qquad f_1' = f_2 = 1,\quad f_4 = 0\,.$$

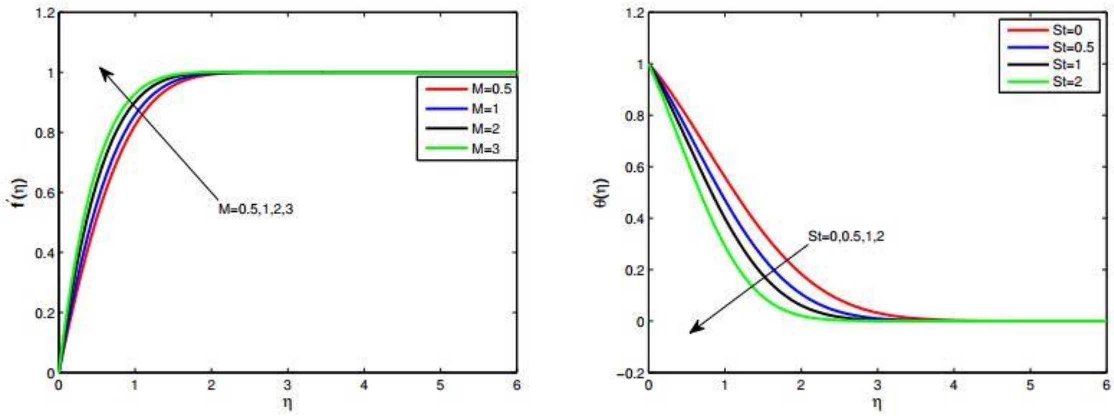
RESULTS

The parameters investigated are: thermal conductivity variation parameter ϵ, viscosity variation parameter α, magnetic parameter M, buoyancy parameter λ, Strouhal number St and the aligned angle Ω. The influence of these parameters on the flow properties are presented by tables 1 and 2, and graphs in figures 2-6.



(a) velocity profiles $f'(\eta)$ (b) temperature profiles $\theta(\eta)$

Figure 2: Effects of varying λ on (a) velocity profiles $f'(\eta)$ and (b) temperature profiles $\theta(\eta)$ for the case when $\Omega = 2\pi/3$, $m = 0.5$, $\epsilon = 0.2$, $\alpha = -0.2$, $Pr = 0.72$, $Ec = 0.2, St = 1, M = 1$.



(a) velocity profiles $f'(\eta)$ (b) temperature profiles $\theta(\eta)$

Figure 3: Effects of varying (a) M on velocity profiles $f'(\eta)$ and (b) St on temperature profiles $\theta(\eta)$ for the case when $\Omega = 2\pi/3$, $m = 0.5$, $\epsilon = 0.2$, $\alpha = -0.2$, $Pr = 0.72$, $Ec = 0.2$, $\lambda = 1$.

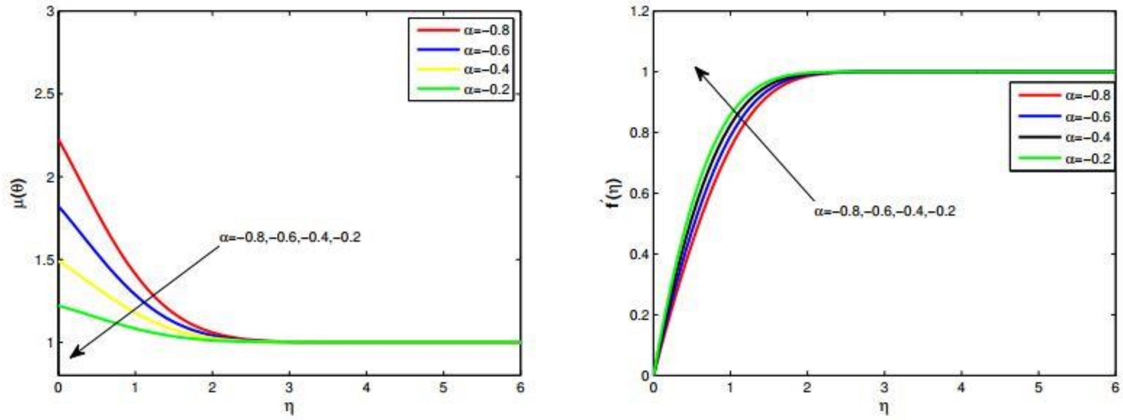
The effect of varying different flow parameters on the skin friction and the heat transfer at the wall is shown in the following tables

Table 1: Skin friction and Nusselt number for different values of parameters

m	ϵ	α	St	M	λ	$C_f(Re_x)^{1/2}$	$Nu_x(Re_x)^{-1/2}$
0	0.2	-0.2	1	1	0.5	2.730545	0.486499
1/3	0.2	-0.2	1	1	0.5	2.948396	0.518212
0.5	0.2	-0.2	1	1	0.5	3.018182	0.533481
1	0.2	-0.2	1	1	0.5	2.32396	0.407623
0.5	0.2	-0.2	1	1	0.5	3.018182	0.533481
0.5	0.2	-0.2	1	1	0.5	2.992264	0.590427
0.5	0.2	-0.2	1	1	0.5	2.960625	0.661035
0.5	0.2	-0.2	1	1	0.5	2.938717	0.709233
0.5	0.2	-0.2	1	1	0.5	3.003422	0.677195
0.5	0.2	-0.2	1	1	0.5	3.025583	0.461154
0.5	0.2	-0.2	1	1	0.5	3.047898	0.242302
0.5	0.2	-0.2	1	1	0.5	3.070367	0.020583
0.5	0.2	-0.2	**0**	1	0.5	3.229174	0.332143
0.5	0.2	-0.2	**0.5**	1	0.5	3.119846	0.439423
0.5	0.2	-0.2	**1**	1	0.5	3.018182	0.533481
0.5	0.2	-0.2	**2**	1	0.5	2.835337	0.695051
0.5	0.2	-0.2	1	**0.5**	0.5	2.589531	0.556885
0.5	0.2	-0.2	1	**1**	0.5	3.018182	0.533481
0.5	0.2	-0.2	1	**2**	0.5	3.734254	0.492627
0.5	0.2	-0.2	1	**3**	0.5	4.335467	0.456943
0.5	0.2	-0.2	1	1	**0**	2.73164	0.533943
0.5	0.2	-0.2	1	1	**2**	3.851212	0.524415
0.5	0.2	-0.2	1	1	**4**	4.918205	0.497476
0.5	0.2	-0.2	1	1	**6**	5.954519	0.456165

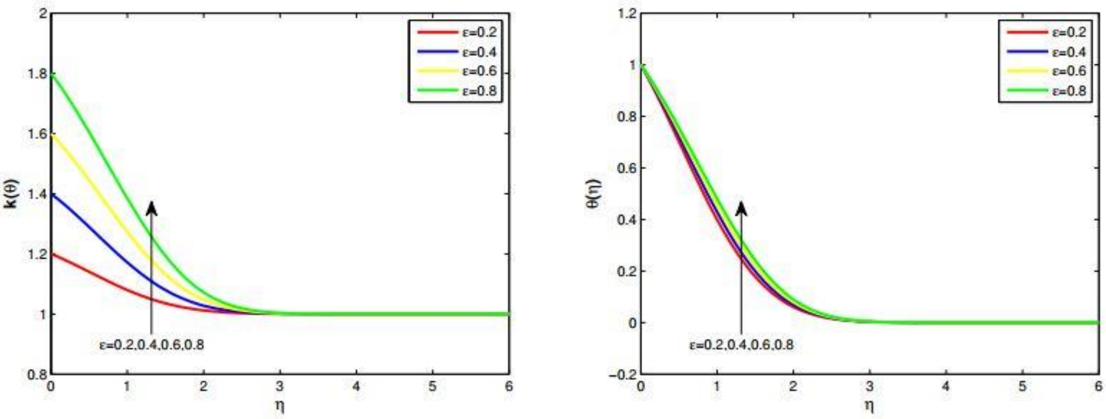
Table 2: Skin friction and Nusselt number for different values of viscosity and thermal conductivity variation parameters

m	ϵ	α	St	M	λ	$C_f(Re_x)^{1/2}$	$Nu_x(Re_x)^{-1/2}$
0.5	**0.2**	-0.2	1	1	0.5	3.018182	0.533481
0.5	**0.4**	-0.2	1	1	0.5	3.029957	0.49104
0.5	**0.6**	-0.2	1	1	0.5	3.040305	0.457927
0.5	**0.8**	-0.2	1	1	0.5	3.049476	0.431194
0.5	0.2	**-0.8**	1	1	0.5	2.050883	0.486136
0.5	0.2	**-0.6**	1	1	0.5	2.338673	0.501885
0.5	0.2	**-0.4**	1	1	0.5	2.660132	0.517691
0.5	0.2	**-0.2**	1	1	0.5	3.018182	0.533481



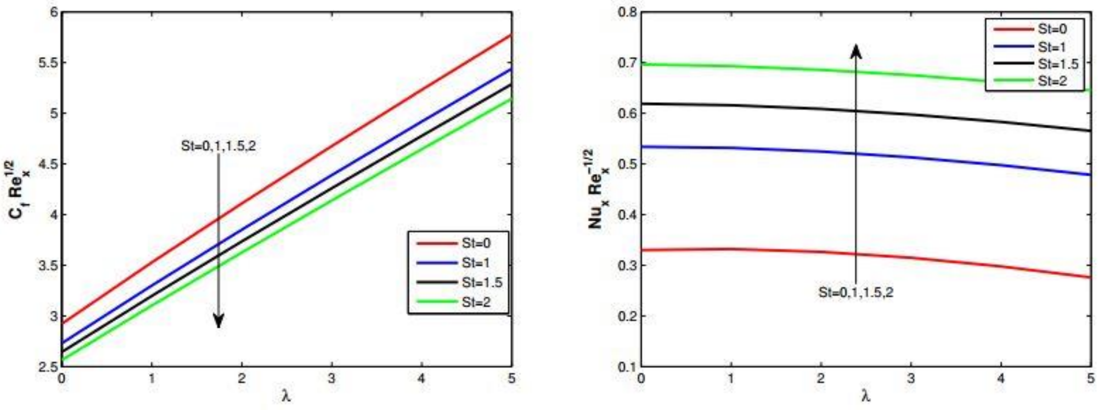
(a) viscosity $\mu(\theta)$ (b) velocity profiles $f'(\eta)$

Figure 4: Effects of varying α on (a) viscosity $\mu(\theta)$ and (b) velocity profiles $f'(\eta)$ for the case when $\Omega = 2\pi/3$, $m = 0.5$, $\epsilon = 0.2$, $Pr = 0.72$, $Ec = 0.2$, $St = 1$, $M = 1$, $\lambda = 0.5$.



(a) thermal conductivity $k(\theta)$ (b) temperature profiles $\theta(\eta)$

Figure 5: Effects of varying ε on (a) thermal conductivity $k(\theta)$ and (b) temperature profiles $\theta(\eta)$ for the case when $\Omega = 2\pi/3$, $m = 0.5$, $\alpha = -0.2$, $Pr = 0.72$, $Ec = 0.2$, $St = 1$, $M = 1$, $\lambda = 0.5$.



(a) Skin friction coefficient $C_f Re_x^{1/2}$ (b) Nusselt number $Nu_x Re_x^{-1/2}$

Figure 6: Effects of varying λ and St on (a) Skin friction coefficient $C_f(Re_x)^{1/2}$ and (b) Nusselt number $Nu_x(Re_x)^{-1/2}$ for the case when $\Omega = 2\pi/3$, $m = 0.5$, $\epsilon = 0.2$, $\alpha = -0.2$, $Pr = 0.72$, $Ec = 0.2$, $M = 1$.

DISCUSSION

The effects of various pertinent parameters on the flow have been analyzed by solving numerically the similarity equations. Discussion of the results follows.

Figures 2(a) and 2(b) show the effect of varying the mixed convection parameter, λ on the velocity profiles, $f'(\eta)$ and temperature profiles, $\theta(\eta)$ respectively. Within the boundary layer, the fluid velocity increases with increasing values of the mixed convection parameter. Velocity overshoots within the boundary layer as mixed convection increases. This is because increasing the value of λ leads to induced favorable pressure gradients which accelerate the flow. This relates to results which were obtained by Roy and Singh (2007). Temperature changes very smoothly within boundary layer as mixed convection increases.

Figure 3(a) displays the effects of varying the magnetic parameter, M on the velocity profile, $f'(\eta)$. The fluid velocity in the boundary layer increases with increasing value of the magnetic parameter. This is because the Lorentz force in the flow direction counteracts viscous forces which leads to thinner velocity boundary layer and hence velocity increases. This relates to results obtained by Das et al. (2015). This increase in velocity produces increase in Skin friction as shown in Table 1.

Figure 4(a) displays the effects of the viscosity variation parameter, α on the viscosity distribution, $\mu(\theta)$. The results depict that viscosity decreases as the viscosity variation parameter, α increases. This compares with the results obtained by Mureithi (2014). A decrease in viscosity causes an increase in velocity and consequently an increase of the Skin friction, C_f as shown in Table 2.

Figure 5(a) displays the effect of thermal conductivity variation parameter, ϵ on the thermal conductivity distribution, $k(\theta)$. The results depict that thermal conductivity increases with increase in thermal conductivity variation parameter, ϵ. This resonates the results obtained by Vajravelu et al. (2013). In Table 2 the Nusselt number, Nu_x decreases as a result of an increase in thermal conductivity variation parameter.

Figure 4(b) shows the effect of viscosity variation parameter, α on the velocity profile, $f'(\eta)$. The fluid velocity in the boundary layer increases with increase in the value of viscosity variation parameter. This is because fluid viscosity decreases as the viscosity variation parameter increases, as already observed in Figure 4(a). This is an important finding in this study because it shows that variability of viscosity has significant impact on the flow velocity.

Figure 5(b) shows the effect of thermal conductivity variation parameter, ϵ on the temperature profile $\theta(\eta)$. The fluid temperature in the boundary layer increases with increasing values of the thermal conductivity variation parameter. This is because thermal conductivity increases as the thermal conductivity variation parameter increases, as observed in Figure 5(a). In this study, variability of thermal conductivity has been found to have significant impact on the flow temperature.

Figure 3(b) depicts the effect of varying the unsteady parameter, St on temperature profile, $\theta(\eta)$. The fluid temperature in the boundary layer is noticeably decreasing due to enhanced temperature flow as the value of the unsteady parameter increases.

Figure 6(a) shows the effect of the mixed convection parameter λ and the unsteady Parameter St on skin friction coefficient $C_f(Re_x)^{1/2}$. Skin friction at the surface of the inclined plate increases as mixed convection parameter increases. This is because the fluid velocity in the boundary layer increases for increasing the values of mixed convection parameter as seen in Figure 2(a). Skin friction decreases as the unsteady parameter increases.

Figure 6(b) shows the effect of the mixed convection parameter λ and the unsteady parameter St on Nusselt number $Nu_x(Re_x)^{-1/2}$. Nusselt number at the surface of the inclined plate decreases as mixed convection parameter increases. This is because the fluid temperature in the boundary layer decreases for increasing the values of mixed convection parameter as seen in Figure 2(b). Nusselt number increases as the unsteady parameter increases.

In summary this study concludes that an increase in viscosity variation parameter, α decreases viscosity which results into increase in velocity. Increasing thermal conductivity variation parameter, ϵ increases thermal conductivity which results into increase in temperature.

This study focused on the theoretical investigation and analysis so it can be used as a stepping stone for future experimental work. Also, future work can be done to include investigation of the effects of suction and injection on a flow over a permeable surface.

ACKNOWLEDGEMENT
The authors appreciate the constructive comments of reviewers and feedback received at the SAMSA 2017 conference

which led to definite improvement of the paper.

REFERENCES

Das S, Jana RN and Makinde OD 2015 Magnetohydrodynamics mixed convective slip flow over an inclined porous plate with viscous dissipation and joule heating. *Alexandr. Engin. J*. **54**:251-261.

Davidson PA 2001 An introduction to magnetohydrodynamics. 1st ed, Cambridge University Press, Cambridge.

Hua H and Su X 2015 Unsteady MHD boundary layer flow and heat transfer over the stretching sheets submerged in a moving fluid with ohmic heating and frictional heating. *Open Physics*. **13**:210-217.

Kafoussias NG and Nanousis ND 1997 Magnetohydrodynamics laminar boundary layer flow over a wedge with suction or injection. *Canad. J. Phys*. **75**:733-745.

Lienhard IV JH and Lienhard V JH 2015 A heat transfer textbook. 4th ed, Phlogiston Press, USA.

Mureithi E 2014 A mixed convection boundary layer flow over a vertical wall in a porous medium with exponentially varying fluid viscosity. *J. Appl. Math. Phys.* **2**: 795-802.

Nazar R, Amin N and Pop I 2004 Unsteady mixed convection boundary layer flow near the stagnation point on a vertical surface in a porous medium. *Int. J. Heat Mass Transf*. **47**:2681-2688.

Raju CSK, Sandeep N, Sulochana C, Sugunamma V and Babu MJ 2015 Radiation, inclined magnetic field and cross-diffusion effects on flow over a streiching suface. *J. Niger. Math. Soc*. **34**:169-180.

Reddy BP 2016 Mass transfer effects on an unsteady MHD free convective flow of an incompressible viscous dissipative fluid past an infinite vertical porous plate. *Int. J. Appl. Mech. Engin.* **21**(1):143-155.

Roy S and Singh PJ 2007. Unsteady mixed convection flow over a vertical cone due to impulsive motion. *Int. J. Heat Mass Transf*. **50**:949-959.

Schlichting H 1979 Boundary layer theory. 7th ed, McGraw-Hill, Inc, USA.

Schnack DD 2009 Lectures in magnetohydrodynamics. 1st ed, Springer Heidelberg, Germany.

Shampine LF, Gladwell I and Thompson S 2003 Solving ODEs with MATLAB, Cambridge University Press, Cambridge.

Vajravelu K, Prasad KV and Ng C 2013 Unsteady convective boundary layer flow of a viscous fluid at a vertical surface with variable fluid properties. *Non-lin. Anal. Real World Applicat*. **14**:455-464.