THE THREE STATE SINGLE SPEAKER MODEL OF SMALL TELEPHONE SYSTEMS

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INTRODUCTION

The speech activity of a single telephone caller may be modelled by either a two-state or a three-state Markov model[1]. If the two state model is used to represent the activity of each speaker in a population of N speakers then the number of active speakers may be represented by a two-dimensional Markov chain[2]. The use of a three state single speaker model results in a three-dimensional Markov chain representation

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Focus attention on the talkspurt inter-arrival times for an aggregate process representing the activity of a population on N speakers. Experimental results[1] have shown that when $N>25$, the talkspurt inter-arrival times for the aggregate process are exponentially distributed thus validating the two state single speaker model. The same experimental results have shown, however, that when $N < 10$ the talkspurt inter-arrival times are not experimentally distributed. This stems from the fact that the silence length distribution for a single speaker is not exponentially distributed[1]. To more accurately model the aggregate process of N speakers when $N<10$, a three state single speaker model which yields non-exponentially distributed silence length distributions has to be used. The three state single speaker model is shown in Fig. 1.

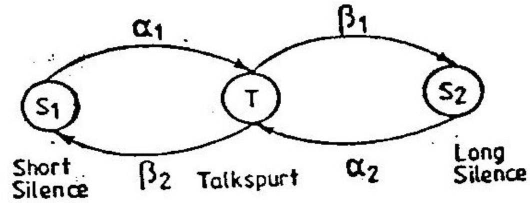

Fig. 1: The Three state single speaker model

Short silences are more frequent than long silences[3,4]. Hence $\beta_1 >> \beta_2$.
We assume that:

(i) all voices conversations begin with a talkspurt

(ii) calls may end even when in silence, but whenever a call ends and a talkspurt is present, that talkspurt must also end.

The two assumptions given above are required to simplify the mathematical equations and do not compromise the accuracy of the resulting model[5].

Given the above assumptions and using standard manipulation with $P_{i,j,k}$ being the probability that i calls are off-hook, j calls are in talkspurt and k calls are in short silence yields the following balance equations for a telephone system with N calls :

$$\lambda P_{i-1,j-1,k} + (i+1-j-k)\alpha_2 P_{1,j-1,k}$$
$$+ (k-1)\alpha_1 P_{i,j-1,k+1} + (j+1)\beta_1 P_{i,J+1,k-1}$$
$$+ (j+1)\beta_2 P_{i,j+1,k} + (i+1)\mu P_{i+1,j+1,k}$$
$$+ (i+1)\mu\delta_{o,j,k} P_{i+1,j,k}$$
$$= \{\lambda[\min(1,N-1)] + (i-j-k)\alpha_2 + k\alpha_1 + i\mu + j(\beta_1 + \beta_2)\} P_{i,j,k}$$

Where $i = 0,1, ..., N$
$j = 0,1, ..., i$
$k = 0,1, ..., i-j$
$\delta_{i,j,k}$ is the Kronecker delta
$P_{i,j,k} = 0$ for $i < 0, j < 0, k < 0, j > i, k > i$, or $i > N$, and
λ = the average call arrival rate, μ^{-1} = the mean call holding time. The traffic intensity, a is defined as
$a = \lambda/\mu$

From the balance equations it can be shown that the ergodic probability that i calls are off-hook, j calls are in talkspurt and k calls are in short silence, $\pi_{i,j,k}$ is given by:

$$\pi_{i,j,k} = \frac{\frac{A^i}{i!}\left(\frac{i}{j+k}\right)\left(\frac{j+k}{k}\right)\left(\frac{\alpha_2}{\beta_2}\right)^{j+k}\left(\frac{\beta_1}{\alpha_1}\right)^k}{\left(\sum_{1=0}^{N}\frac{A^1}{1!}\right)\{\frac{\alpha_2}{\beta_2}\left(1+\frac{\beta_1}{\alpha_1}+\frac{\beta_2}{\alpha_2}\right)\}^i}$$

Note that the balance equations can be used to obtain a three-dimensional Markov chain representation of the process.The state space of the Markov chain has (1/6)(N+1+(N+2)(N+3) states.Thus even for small systems with N=10, say, the state space is fairly large at 286.

CONCLUSION

This note has developed a Markov chain representation of small telephone systems. The resulting model has a fairly large state space for even very small systems.

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