APPLICATION OF OPTIMUM DESIGN RESULTS FOR SPRING UNDER DIFFERENTIAL DESIGN REQUIREMENTS

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ABSTRACT

There have been many experts and scholars who have contributed their efforts in the study of optimum design of springs during recent years. The optimum design of spring has virtually raised the design quality Due to the differences of objective conditions, it often requires to carry out optimum design of different springs under different design requirements. It is obvious that it is a complex problem. How could it be possible to maximally meet the actual needs of various design requirements and simultaneously simplify optimum design?

This article refers to an example of optimum design of a cylinder spring with compressed spirals, and according to the similarity theory to apply its optimum result, through a very simple calculation, to another optimum design of spring under different design requirements.

INTRODUCTION

There have been many experts and scholars who have contributed their efforts in the study of optimum design of springs during recent years.The optimum design of spring has virtually raised the design quality Due to the differences of objective conditions, it often requires to carry out optimum design of different springs under different design requirements. It is obvious that it is a complex problem. How could it be possible to maximally meet the actual needs of various design requirements and simultaneously simplify optimum design?

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Examples

A practical example is shown below to introduce a mathematical model of the optimal design of spring and its optimal result.

Example 1:

Try to design cylinder spring with compressed spirals. Its deformed length $\lambda = 16.59$ mm. The maximum working temperature is 150 ^{o}C, material required is chromium vanadium steel 50CrVA. The desired working life is 10^5 cycle index. After intensified treatment, its allowable shearing stress $[\tau] = 404.9$ MPa. The required spring rate c = 41 N/mm. The axial force that the spring endures F = 680.2 N. Try to determine the wire diameter d, and mean diameter of the spring D, and number of effective coils n. It is requested to design, under the conditions to meet the strength and rigidity requirements, a structure of minimum weight.

Solution 1:

Write out the optimum mathematical model of this example.

Design variables: Design variables are the wire diameter d, mean diameter of the spring D, and number of coils n.

Therefore

$$X = [x_1,x_2,x_3]^T = [d,D,n]^T \tag{1}$$

Objective function:

Taking the number of noneffective coils $n_1 = 1.8$, weight target of the spring to be calculated by its volume is:

$$V = \frac{\pi^2}{4} d^2 Dn + \frac{\pi^2}{4} d^2 Dn_1 \tag{2}$$

$$f(X)=\frac{\pi^2}{4}d^2Dn+\frac{\pi^2}{4}d^2Dn_1=2.47x_1^2x_2x_3+4.44x_1^2x_2 \tag{3}$$

Design constraints:

Based on the strength requirement of the spring,

$$\tau=\frac{8k_1FD}{\pi d^3}\le[\tau] \tag{4}$$

in which k_1 (curvature coefficient)

$$k_1=\frac{4C-1}{4C-4}+\frac{0.615}{C}=\frac{1.6}{\left(\frac{D}{d}\right)^{0.14}} \tag{5}$$

C = spring exponent , $C=\frac{D}{d}$

F = axial force endured by the spring, N.

From this computation we can have:

$$g_1(X)=\frac{8k_1FD}{\pi d^3[\tau]}-1=8\cdot\frac{1.6}{\left(\frac{D}{d}\right)^{0.14}}\cdot\frac{FD}{\pi d^3[\tau]}-1$$

$$=6.84x_1^{-2.86}x_2^{0.86}-1\le 0 \tag{6}$$

Based on the rigidity requirement of the spring

$$n=\frac{Gd^4}{8cD^3} \tag{7}$$

Where, G shearing elastic modulus of the spring material, and its value here is 8×10^4 MPa, From this, we can have

$$g_z(X)=\frac{Gd^4}{8cD^3n}-1=243.90x_1x_2^{-3}x_3^{-1}-1\leq 0 \quad (8)$$

According to experience,

$$4\leq\frac{D}{d}\leq 14 \quad (9)$$

From this, we can have

$$g_3(x)=4-\frac{x_2}{x_1}\leq 0 \quad (10)$$

$$g_4(x)=\frac{x_2}{x_1}-14\leq 0 \quad (11)$$

From the analysis shown above, it is known that the optimum mathematical model of this example is

$$f(X)=2.47x_1^2x_2x_3+4.44x_1^2x_2 \quad (12)$$

subject to $g_1(X)=6.84x_1^{-2.86}x_2^{0.86}-1\leq 0$ (13)

$$g_2(X)=243.90x_1^4x_2^{-3}x_3^{-1}-1\leq 0 \quad (14)$$

$$g_3(X)=4-\frac{x_2}{x_1}\leq 0 \quad (15)$$

$$g_4(X)=\frac{x_2}{x_1}-14\leq 0 \quad (16)$$

$$x_1,x_2,x_3\geq 0$$

Optimization of Design Result

When the spring is optimized by optimal method (see the note) its optimal solution is:

$X^*=[d\ D\ n]^T=[5\ 20\ 20]^T$ (17)

SIMILARITY CRITERION CALCULATION

Equation analysis method is adopted to calculate the similarity Criterion

The Method of Equation Analysis[2]

The physical or geometric relations in the system can often be expressed in form of equation. When the two phenomena are similar, the form of equation should be entirely the same, and the ratio of any two corresponding items should be equal. For example, from the concept of geometric similarity we know that if two triangles are similar, its corresponding side is proportioned, and the ratio is equal.

The steps of equation analysis method are as follows:

(i) Write out the equations of physical or geometric relations

(ii) Divide remained items using any of item in equation

(iii) Each derivative involved in equation be replaced by the corresponding ratio. i.e. dy/dt replaced by y/t.

(iv) To add the short of the basic equation in condition of single value (i.e. Change the common solution of phenomena into special solution n certain condition), then set up new equations to determine other similarity criteria.

(v) Corresponding parameters in each ratio be replaced by the corresponding similarity ratio, making it equal to 1, then each relation formulae obtained are the similarity targets between two similar phenomena.

Calculation of Similarity Standards

When calculating similarity standards, in order to make it simple and clear, it is easy to neglect the secondary and less important physical sizes. For example the bearing spiral of the spring is generally 1.5, and its spiral angle is not large, therefore when the number of working cycles n is comparatively bigger, the calculation of column compressed spiral spring volume may use the total spring cycles na to replace n of the working cycle.

(i) The wight target of the spring is calculated by its volume.

$$V = \frac{\pi^2}{4} d^2 D n_a = 2.47 d^2 D n_a$$

That is $V - 2.47 d^2 D n_a = 0$ (18)

To divide all items of the equation with the 2nd item of the equation. The result is

$$\frac{V}{2.47d^2Dn_a}-1=0$$

Then the first similarity standard can be obtained as

$$\alpha_1=\frac{V}{d^2Dn_a} \tag{19}$$

(ii) Based on the strength condition of the spring, it is known from equation (4)

$$\frac{8k_1FD}{\pi d^3}-[\tau]=0 \tag{20}$$

Putting equation (5) into (20), the result is

$$\frac{4.07FD^{0.86}}{[\tau]d^{2.86}}-1=0$$

From this, the similarity standard can be obtained as

$$\alpha_2=\frac{FD^{0.86}}{[\tau]d^{2.86}} \tag{21}$$

(iii) Based on the rigidity condition of the spring, take equation (7) to obtain similarity standard with the steps mentioned above

$$\alpha_3=\frac{Gd_4}{cD^3n_a}$$

The shearing elastic modulus can be considered to be a constant, then

$$\alpha_3=\frac{d_4}{cD^3n_\alpha} \tag{22}$$

There are 6 physical parameters in this example: V,d, D,n, F,[ ] in which there are three basic physical dimensions: [F],[L], [T}. Therefore the number of similarity standards is 3, the items obtained above is just equal to this. Therefore the procedure to determine solution is now ended.

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Calculation of Similar Targets

It is known from similarity principle, as to similar phenomena, that its targets are equal to 1, therefore equations [19], [21] and [22] can be written as similar targets respectively

$$T_1 = \frac{C_v}{C_d^2 C_D C_{na}} = 1 \tag{23}$$

$$T_2 = \frac{C_F C_D^{0.86}}{C[\tau] C_d^{2.86}} = 1 \tag{24}$$

$$T_3 = \frac{C_d^4}{C_c C_D^3 C_{na}} = 1 \tag{25}$$

THE OPTIMAL DESIGN OF SPRING WITH DIFFERENT DESIGN REQUIREMENTS

According to the similarity standards or similarity tagets obtained above, you can use optimal result in Example I through simple calculation, to get directly the optimal design parameters of the spring under diferent design requirements.

Example 2

Try to design a column compressed spiral spring, its deformed length = 20 mm, the axial compression force F' = 400 N, the size of structure that limits the mean diameter of the spring D' ≤ 23 mm, other conditions are the same as in Example 1. Try to determine the diameter of the wire d'and working cycles n'. it is required to have the lightest structure draft under the conditions to satisfy the strength and rigidity conditions.

Solution: It is known from the problem that the spring rate c' is

$$c' = \frac{F'}{N} = \frac{400}{20} = 20N/mm \tag{26}$$

Suppose D' is 23 mm, then with D=20 mm from example I (see equation 17)

$$C_D = \frac{D'}{D} = \frac{23}{20} = 1.15 \quad (27)$$

Again $C_F = \frac{F'}{F} = \frac{400}{680.2} = 0.5881$ (28)

$$C_{[\tau]} = \frac{[\tau]}{[\tau]} = 1 \quad (29)$$

$$C_c = \frac{C'}{C} = \frac{20}{41} = 0.4878 \quad (30)$$

From equation (24), you can obtain

$$C_d = C_F^{0.3497} C_D^{0.3007} = 0.5881^{0.3497} X1.15^{0.3007} = 0.8662 \quad (31)$$

$$d' = dC_d = 5X0.8662 = 4.331mm \quad (32)$$

From equation (25), you can obtain

$$C_{na} = \frac{C_d^4}{C_c C_D^3} = \frac{0.8662^4}{0.4878X1.15^3} = 0.7588 \quad (33)$$

$n_a = nC_a = 20 \text{ x } 0.7588 = 15.176$ (34)

After rounding off, d is taken as 4.5 mm and n' as 16. But now it violates the constraints in particular. If n' is taken as 1, the constraints can be satisfied.

Therefore the optimal solution obtained by using similarity theories is as follows:

$$X^* = [x_1 x_2 x_3]^T = [dDn]^T = [4.52317]^T \quad (35)$$

If the design in Example 2 can be optimized by using optimal method, the best parameters obtained are the same as the figures above[1].

CONCLUSION

The theory and examples indicate that, after a spring has been optimized in design, according to the similariy theory, this result of optimal design can be transformed into other optimal design parameters of the spring under different design requirement through simple calculation, which may greatly reduce the workload of design. This is very practical and convenient to the broad engineers.

Note: This article chiefly refers to the application of similarity theory to apply the result of optimization of spring to other optimal design of spring under different requirements. Therefore the process of optimal design for the spring is omitted.

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