ESTABLISHMENT OF LIMITS TO THE PRINCIPAL DIMENSIONS OF THE ROTOR OF A HIGH SPEED HIGH POWER DENSITY ELECTRICAL MACHINE AS AFFECTED BY THE MECHANICAL LIMITS

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ABSTRACT

Mechanical constraints responsible for setting limits to the principal dimensions of the solid rotors of High Speed Electrical Machines (HSEM) are discussed. A methodological approach for establishing the dimensional limits of the solid rotors of HSEM is presented.

On the basis of two simplistic models of solid rotors, typical curves relating the rotor's dimensional limits to the values of Very High Speeds (VHS) and also the dependence of the dimensional limits on the mechanical properties of the rotor materials are shown. The influence of the topological changes of the solid rotors to the dimensional limits is also shown.

INTRODUCTION

Recent years have witnessed a growing interest in HSEMs due to their numerous advantages. Amongst the merits of HSEM over their conventional speed counterparts are their higher power densities, i.e. smaller machine sizes for the same power rating or higher power densities for the same size of machine frame. Conventional speed electrical machines are those which run off or feed to the grid with standard industrial frequencies of $f = 50\ Hz$ or $f = 60\ Hz$.

The designing process of the HSEM converges two major parts or paths which are based on two types of constraints namely, the constraints of electromagnetic nature and those of mechanical nature. This paper is however limited to treatment of only the constraints of mechanical nature which are increasingly important for the safety of the HSEM as higher power densities are sought from them.

The rotating parts of the reduced in geometrical sizes HSEM are subjected to increased centrifugal forces as the rated speeds of the machines are increased. While, for a given rated power, the reduction in size contradicts with the requirement for an increased bore volume in pursuing a HSEM of higher power density, the mechanical properties of the solid rotor material determines the radial dimensional limits of the rotor for a given topology. Thus, in order to facilitate a trade-off between the rated speed n (rpm) and the bore volume D^2L_l, the designer of high power density HSEM needs to establish also a domain in which he can dimension such a machine for a given rotor material.

It should be noted that massive rotors made of soft magnetic steels are implemented in HSEM due to their superiority in mechanical strength despite their magnetic inferiority to the rotors of laminated electrical steel. A massive rotor can be machined out of a steel forge of the soft magnetic material to obtain the required magnetic topology. It can also be realised out of composite materials which are in powder form by pressing together the insulated iron particles, using a method called Hot Isostatic Pressure (HIP), into a monolith of the required topology. The focus of this paper is on the solid rotors machined out of steel forges.

Note that VHS in this work refers to rotational speeds in the range of 20 000 rpm up to 100 000 rpm.

METHOD

1. Determination of the rotor's limit diameter

From the basic relationship of an ac machine shown in *eqn. (1)* below, the apparent power of a HSEM depends on parameters of electromagnetic nature and those of mechanical nature as:

$$S_n = C_m D^2 L_l\, n \qquad (1)$$

where the bore diameter D, the active length L_l and the rated rotational speed n are the mechanical parameters. The machine constant C_m

$$C_m = \frac{\pi^2 10^{-7}}{60} k_{w1} k_B \alpha_\delta A_1 B_\delta \qquad (2)$$

comprise of the electromagnetic parameters namely:

A_l the armature current loading, A/cm; B_δ the peak value of the air gap flux density, T; k_{wl} the winding factor; k_B the emf form factor and α_δ the pole span factor

The mechanical parameters can be jointly looked at in the context of the kinematic relation of the rotating rotor. For virtually all types of rotary machines, the peripheral speed (or tip speed) of the rotor is the design/characteristic/operational parameter rather than the rotational speed n. The peripheral speed, which is obtained as:

$$u = \pi D \frac{n}{60} \qquad (3)$$

is defined by optimum operating conditions and /or mechanical limitations. As regards the peripheral speed of the HSEM rotor, some authors such as (Fuchs, 1983) and (Larjola, 1988) treated the sonic speed as the practical limit beyond which the HSEM rotors couldn't be safely operated.

This question was answered by (Tapani, 1993) when presenting a HSEM which attains a supersonic peripheral speed when running at its rated speed. He emphasised on the requirement of a strong and, therefore, a mechanically sound rotor construction capable of withstanding the stresses in the machine, particularly at the rated speed of operation.

For a given rotor material, the HSEM's dimensioning domain will be the one under the $D_{lim} = f(n_{mr})$ curve of the recommended limit diameter D_{lim} at a given speed of rotation which is correspondingly taken as the maximum rated mechanical speed n_{mr}

The $D_{lim} = f(n_{mr})$ curve is characterised by the mechanical properties of the rotor material and is obtained with consideration of the enormous magnitudes of the centrifugal forces which acts on the rotors when rotational speeds are increased. The mechanical properties of the material are considered by the value of the maximum allowable yield stress which is usually obtained as $\sigma_m = \xi \cdot \sigma_s$ whereby factor ξ, which can be taken anywhere from $\xi = 0.5$ up to $\xi = 0.7$, is introduced so as to consider the presence of impurities in the material. The maximum yield stress of the material σ_s is given by the manufacturer. The limit rotor diameter can then be found (Wiart, 1982) as:

$$D_{lim} = \frac{2}{\Omega_{mr} k_1} \sqrt{\frac{\sigma_m}{C \rho_R}} \tag{4}$$

where, ρ_R is the density of the rotor material, $C = (3+\nu)/8 = 0.41$ is a coefficient obtained for steel whose Poisson's coefficient is approximately $\nu = 0.3$.

$\Omega_{mr} = \dfrac{2\pi n_{mr}}{60}$ is the maximum rated mechanical angular speed

$k_1 = \dfrac{n_m}{n_{mr}}$ is an overspeed ratio

n_{mr} is the highest expected mechanical overspeed of the rotor

The overspeed ratio may be taken above unity, about $k_1 = 1.2$. This implies that an overspeed of up to 20% have to be accommodated when dimensioning a solid rotor of a given material. This serves as a rotor's mechanical safety margin.

Thus, at a given rated power P_2 and a rated speed n, the maximum diameter D_{lim} of the given rotor material will be mainly a function of the mechanical properties. The minimum diameter of such a rotor, which is mostly the shaft diameter, will be characterised by the torque requirement of the machine. According to (Levi, 1984), the shaft diameter can be determined as :

$$D_{sh} = 0.012 + 0.176 \cdot \sqrt[3]{\frac{P_2}{n}} \tag{5}$$

where P_2 is the rated output power in kW and n is the rated speed.

Note: For the case in *eqn (4)* above, the rated speed will thus be n_{mr}.

DETERMINATION OF THE TOTAL LENGTH OF THE ROTOR

By the rotor's total length reference is made to the entire axial length between the two bearings on which the rotor is suspended. With consideration of the actual topological features, the rotor has to be dimensioned such that all its critical speeds are far enough from the operating speed. For the case where some critical speeds are below the operating speed, or if we have an adjustable speed HSEM situation then, means have to be provided to ensure a fast passage of the critical speeds. This ensures the safety of the rotor against some severe and detrimental vibrations which takes place when the rotor is left to run at its critical speeds. The corresponding order numbers ($k = 1, 2, ...$) of the critical speeds will change by effecting some geometrical (topological and dimensional) changes to the dimensioned rotor. Canning of the rotor, particularly a salient-pole rotor, is one such way by which the some geometrical changes can be realised on a dimensioned rotor. Rotor-canning improves not only the aerodynamic characteristics of the rotor but it also the rigidity. However, retention of the canning structure on the high speed rotor is problematic and the involved costs may be prohibitive.

An alternative way of ensuring the rotor's safety from its critical speeds can be dealt with at the designing stage of the HSEM.

As the rotor's dimensions and topology are usually determined by the electromagnetic requirements of the machine, little room is left for altering the values of the critical speeds by changing the rotor's dimensions. This is particularly emphasised in the case when, for a given rotor material, a HSEM of very high power density is sought. Knowledge of the rotor's critical speeds at the design stage is important as it allows to re-dimension the rotor, particularly its active length L_l, should a need arise. In such a case, the rotor is redimensioned whereby the machine constant C_m is changed accordingly so as to satisfy *eqn. (1)* which is governed by the design specifications. For simplicity, the active length L_l can be obtained through an aspect ratio which relates the length L_l to the stator's bore diameter D as $\lambda = L_l/D$. Note that the bore diameter may be taken as the same as the rotor diameter since the machine's air-gap length is usually very small. Depending on the type of HSEM, the aspect ratio may vary from a fraction to a number above unity. So, having obtained the radial and axial dimensions of the solid rotor, critical speeds of the rotor system are calculated and check against the operating speed.

Using two simple models of solid rotors for HSEM, the method for obtaining the critical speeds is shown below. In the method, the length and the topological features of the rotor are considered. The two models, which are shown in Figs 1a and 1b below, differ in topology, and hence their flexibility, but have the same D_{lim} and total length L between the bearings. Figs. 2a through 5 show some typical results as influenced by the change of rotor topology, length and even the "change" of mechanicl properties of the rotor material.

The model M-2 is complex than M-1. In fact, the actual topologies of the various rotors used in different types of HSEM are complex than the one shown in Fig. 1b.

As noted earlier, rotors of different types of electrical machines, including solid rotors for the HSEM, differ in configurations depending on the requirement of their magnetic circuits as well as their mechanical requirements.

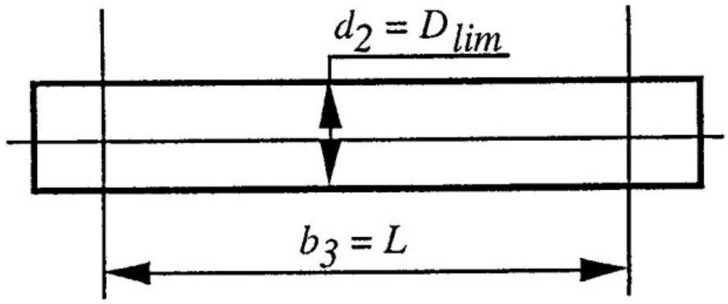

Fig. 1a Model M-1 a constant-diameter solid rotor

To emphasise on complexity of the rotor's topologies one can take the classical rotor of a homopolar machine whose dimensions changes in both the radial and axial directions throughout the total length L.

It is these topological complexities of the rotors that require an accurate mathematical representation for accurate determination of the critical speeds of the rotor systems. A formula by (Hutoresky et al, 1987) is applied in determining the k-th critical speeds of the rotor models. With

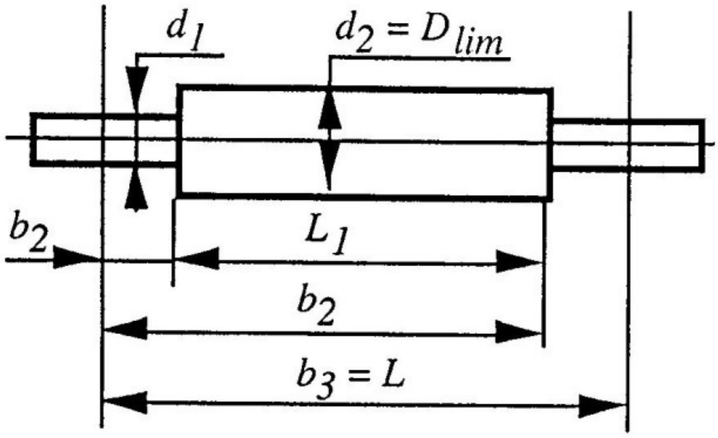

Fig. 1b Model M-2 a graduated solid rotor

an assumption that the rotor is held on two infinitely rigid bearings, its k-th critical speeds can be determined as:

$$n_k = \frac{\pi^2 k^2}{2\pi L^2}\sqrt{g \cdot E \cdot \frac{J'}{q'}} \tag{6}$$

where, $k = 1, 2, ..$ an order number of critical speed, L the total length between the bearings, mm,.

g the acceleration due to gravity, 981 cm/s^2

E the Young's modulus of elasticity, N/mm^2

In order that *eqn. (6)* could be used on any model, a reference point for the co-ordinates has to be fixed on either of the assumed two rigid bearings. Then, b_i co-ordinates are obtained for each of the parts of the rotor having a constant diameter d_i as seen in the models in Figs. 1a and 1b. For the rotor parts $i = 1, 2..,$ unitless co-ordinates x_i are obtained as:

$$x_i = k\frac{b_i}{L} \tag{6a}$$

their corresponding aiding functions are calculated as:

$$F(x_i) = x_i - \frac{\sin(2\pi x_i)}{2\pi} \quad (6b)$$

whereby their Δ changes are obtained as:

$$\Delta_i = F(x_i) - F(x_{i-1}) \quad (6c)$$

The mass per unit length of each constant-diameter part is calculated as:

$$q_i = \frac{\pi d_i^2}{4} \rho_R \quad (6d)$$

and the moment of inertia of every constant-diameter part of the rotor is obtained as:

$$J_i = \frac{\pi d_i^4}{64} \quad (6e)$$

The parameters J' and q' in *eqn. (5)* are obtained as:

$$J' = \left(\frac{1}{k} \sum_{i=1}^{m} \frac{\Delta_i}{J_i} \right)^{-1} \quad (6f)$$

$$q' = \frac{1}{k} \sum_{i=1}^{m} \Delta_i \cdot q_i \quad (6g)$$

For the simplest case of the rotor (model M-1), the above described method can be used. However, for such a simple case of M-1, one can also directly determine a suitable length L with respect to a particular k-th critical speed as:

$$L = \frac{D_{\lim}}{2} k\pi \sqrt{\frac{k_1}{k_2}} \sqrt[4]{\frac{C \cdot E}{4 \cdot \sigma_m}} \quad (7)$$

where $k_2 = n_k/n_{mr}$. This ensures that the k-th critical speed value is sufficiently away from the rated n_{mr} speed. If the rotor diameter and the rated speed are below the $D_{lim} = f(n_{mr})$ curve (see Fig 2), then the factor of $0.9 > k_2 > 1.1$ is usually recommended.

Calculations

A spreadsheet technique was used in the calculations. Both the M-1 and M-2 rotor models were assumed to be of the solid magnetic steel, material type:

Steel type	40 NiCrMo 4
Young's modulus E	216 GPa
Density	ρ_R 7850 kg/m^3
Yield stress	σ_s 700 MPa

For simplicity of dimensioning model M-2, the rotor's active length was taken as $L_1 = 0.7{\cdot}L$ and the diameter $d_1 = 0.4.D_{lim}$ where $d_2 = D_{lim}$. A factor $k_1 = 1.2$ was used.

RESULTS AND DISCUSSIONS

Fig. 2 shows a curve of the limit diameter D_{lim} as a function of the maximum rated speed n_{mr} for the solid rotor material 40NiCrMo4 and calculated at $\xi = 0.5$. This curve will remain the same regardless of the topology of the rotor unless there is an enhancement of the material's mechanical properties.

A plot in Fig. 2a comprises of several curves of the type seen in Fig 2 whereby the factor ξ is varied. The variation of factor ξ has the same effect as changing the mechanical properties of the rotor material. Likewise, enhancement of the mechanical properties of the rotor material will result with a similar plot as the one in Fig. 2a. In such a case, the designer of a high power density HSEM will be confined to a domain under a typical plot like the one in Fig. 2a. If one chooses to look at Fig. 2a in terms of the peripheral speeds, Fig. 2b will be a logical inference of what has been said about Fig. 2a whereby the design domain changes with the mechanical properties of the rotor material.

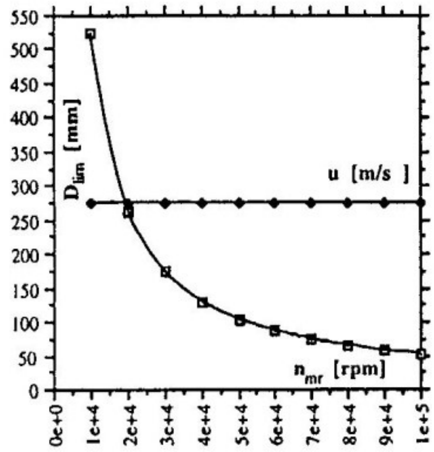

Fig. 2 Limit diameters D_{lim} as functions of max. speed n_{mr} at speed factor factor ξ=0.5 thus u = 275 m/s

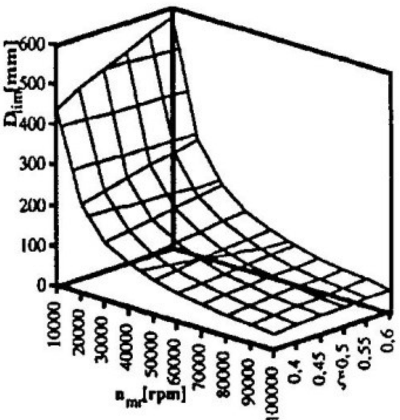

Fig. 2a Limit diameter D_{lim} as functions of max rated speed n_{mr} for varying ξ factor

Fig 3 shows curves of critical speeds of an M-2 model of a solid rotor material 40NiCrMo4 for several values of its predetermined limit diameters. The shaded part in Fig. 3 is a speed band from which the critical speeds of the rotor system ought to be kept away when ultimate but safe use of the mechanical propertries of the 40NiCrMo4 rotor material is sought. The curve $D_{lim} = f(n_{mr})$ is not seen in Fig. 3 and Fig 4 as it passes right in the middle of the shaded band, a prohibited speed band for the rotor's critical speeds. It should be noted that the part above the curve $D_{lim} = f(n_{mr})$ will be redundant to the designer of HSEM for a given 40NiCrMo4 rotor material. However, for the same rotor material and topology, these Figs 3 and 4 will be representing some typical curves and all the curves would be relevant if curve $D_{lim} = f(n_{mr})$ is substituted by $D = f(n)$ whereby D and n are some correspondingly lower values in the rotor material's design domain.

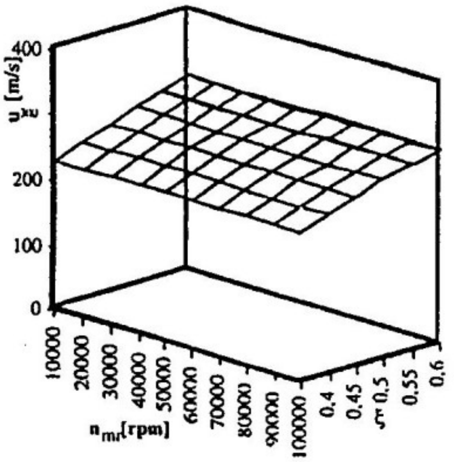

Fig. 2b Rotor's peripheral speeds foe varying ξ factors

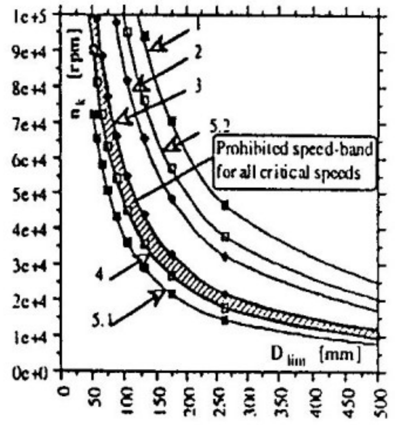

Fig. 3 k-th critical speeds of M-2 as functions of D_{lim}

While curves 1, 2, 3 and 4 are of the first critical speeds ($k = 1$) for $L = \lambda D_{lim}$ obtained at $\lambda = 2.5, 3, 3{,}6$ and 4 respectively, the curves 5.1 and 5.2 are respectively of the first and second critical speeds, (i.e. the case where $k = 1$ and $k = 2$) with $\lambda = 4.5$. The last case, i.e. when $\lambda = 4.5$, represents a typical case of the rotor whose operational speed is between two critical speeds. Note again, curve *5.2* is redundant for the case of the $D_{lim} = f(n_{mr})$.

Thus, from Fig. 3, a logical inference can be made that the rotor becomes more flexible and thus meets its critical speeds much earlier as its total length L is increased at the same radial dimensions and the rotor topology.

Fig. 4 compares the critical speeds of the two models, i.e. M-1 and M-2 having the same D_{lim}. It is seen that, at the same order of critical speed, model M-1 exhibits higher values of critical speeds than model M-2. This helps the above made inference on the flexibility, and thus a suitable length L, of the rotor as its topology or configuration changes.

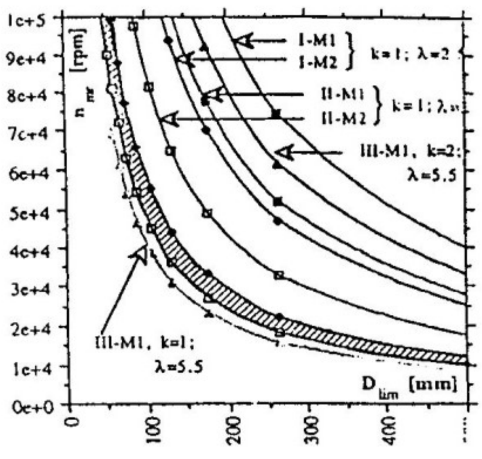

Fig. 4 Comparison of M-1 and M-2 on their k-th critical speeds for the same D_{lim}

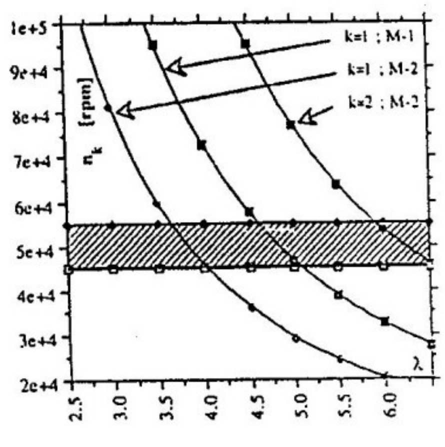

Fig.5 Restricted Values of λ of M-1 and M-2 at 50,000 rpm.

Fig. 5 compares once again M-1 to M-2 with regard to their corresponding *k*-th critical speeds. It also shows some restricted values of λ with respect to a particular *k*-th critical speeds. This is typical for a given rotor material, rotor topology and a predetermined rotor's radial dimensions.

CONCLUSION

With accurate mathematical modelling of any topology of a solid rotor, reliable mechanical limits in dimensioning a HSEM can be established for a given type of rotor material. This helps to identifying a domain in which the designer of HSEM will work. As the design process is an iterative one, it also helps to reduce the design time in the cases where higher power density HSEM are sought. Such information may also serve as a useful feedback to the manufacturers of the rotor material for improvement of their products.

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