SIMPLE ELASTO-PLASTIC LIMIT ANALYSIS OF PLANE FRAMES SUBJECTED TO COMBINED BENDING MOMENT AND EXIAL FORCE

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ABSTRACT

An algorithm for simple elasto-plastic limit analysis of plane frames subjected to combined bending moment and axial force actions is presented. The technique takes into account the influence of axial forces on the ultimate load carrying capacity of the structure retaining the assumption of small displacement. The method allows for unloading of plastic hinges.

The algorithm has been programmed in a program Elasto-Plastic Limit analysis of plane frames (EPLAF). It gives the location and sequence of formation of plastic hinges plus the corresponding load factors at which the hinges are formed, collapse loads, nodal displacements, member actions, support reactions and plastic hinge rotations.

The applicability of the procedure has been demonstrated through analysis of Frame-A. The numerical solutions of the frame are compared to those obtained in the traditional simple elasto-plastic analysis which neglects the effects of axial forces on the ultimate load carrying capacity of the structure.

The numerical solutions obtained showed that when the amount of axial force in the elements of a frame exceed 15% of the respective element plastic axial force, the conventional simple order elasto-plastic limit analysis method over estimates the load carrying capacity of structures, grossly mispredicts the bending moment distributions in the structure at full plastic stage. Further, the new technique increased the rate of structure stiffness degradation and altered the number, location and sequence of formation of plastic hinges and the type of collapse mechanism mode when axial forces were large.

INTRODUCTION

Simple elasto-plastic limit analysis is the process of determining the ultimate load capacity of a structure, the collapse mechanism mode and forces & moment distributions in collapse mode due to specific and most unfavourable loading configurations allowing for first order deflections only. the simple analysis of plastic structures under bending moment action only and based on simple plastic theory was developed in the 1950 - 1980 decades. In the development phase of the plastic theory it was recognized that axial forces had effects on the structural behaviour of plastic frames [1]. This phenomenal made practical application of plastic theory to frames subjected to heavy axial forces very limited. This limitation being attributed to complexities brought about by non-linearities of material yield conditions and the iterative procedures required by the upper and lower bound solutions.

LITERATURE REVIEW

The characteristic of simple plastic limit analysis is the determination of magnitude of ultimate load which will render a structure unfit for use. Beedle (1966)[2], Horne (1979)[3] and Neal (1977)[4] have presented Mechanism, Incremental, Pseudomoment distribution and Statical methods for predicting this ultimate load of redundant frame assuming constant section plastic theory, are the simplest available procedures for predicting the ultimate load carrying capacity of frames. In these procedures virtual work equations based on an intuitively assumed collapse mechanism mode, which can be correct or erroneous, are formulated and solved. Generally trial-and-error procedures are done to determine the true collapse load and the collapse mechanism mode.

Adeli and Chyou (1987)[5] and Watwood (1979)[6] used kinematic approach with automatic generation of independent mechanisms to determine the collapse load of low-rise frames of general configuration. Another procedure based on the static approach which was proposed by Wang (1963)[7] and later modified by Harrison (1979)[8], Sudhakar (1986)[9] and Monasa (1990)[10] can be used to determine the collapse load of general planar frames. The methods provide step-by-step procedures by which the entire load-deflection response of complex frames can be determined while avoiding the difficult task of identifying the correct collapse mechanism through trial and error methods. In these methods, plastic hinges are formed sequentially in the frame members, and the corresponding load factors are determined until the collapse mechanisms is reached. The procedures proposed by Adeli (1987), Watwood (1979), Wang (1963), Harrison (1979), Sudhakar (1986) and Monasa (1990) however assumed constant section plastic moment by disregarding the effects of axial forces. By using both linear and non-linear mathematical programming techniques, Cohn (1974) proposed procedures for analysing plastic structures subjected to combined flexural moment and normal forces.

THE RESEARCH PROBLEM

Simple plastic theory neglects influence of axial on structural behaviour of structural plane frames. This simplification leads to results which do not reflect true structural behaviour of the frames. The deficiency resulting from the theory is illustrated by the following case.

A plane frame shown in Fig. 1 has been analysed by Simple elasto-plastic limit analysis method under consideration of flexural moments only. The full range load-displacement of node C of the frame is plotted in curve I of Fig. 2. The same system has been tested by Majid [1972] and reported by Kassimali [1983]. Their experimental results are plotted in curve II of Fig. 2.

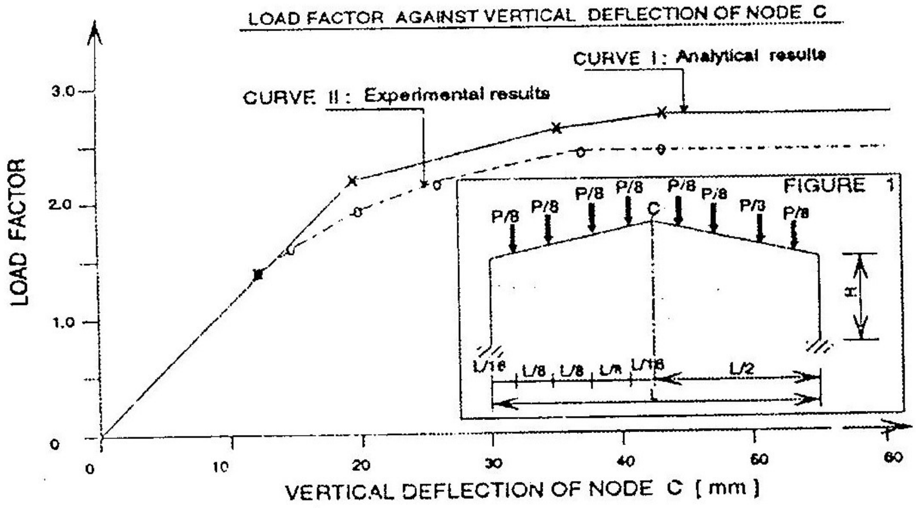


Fig. 1(INSERT) **Portal plane frame: Shape and loading configuration**
Fig. 2 **Load factor against vertical deflection of node C**

Geometrical and structural properties of portal plane frame shown in Fig. 1:

$EI = 460748$ kN-mm^2 $\quad$ $M_{pl} = 153.793$ kN-mm

$L = 1219$ mm $\quad$ $H = 406$ mm $\quad$ $A = 169$ mm^2

The analytical and experimental ultimate loads results were 2.729 and 2.469 kN respectively. This shows a discrepancy of 11% in ultimate load in the two results. At the same time the analysis indicated that the structure was stiffer from service load to collapse. Thus there exist some discrepancy between experimental and analytical results based on simple plastic theory.

With the rigorous progress in computer technology it is possible to develop technique and numerical solutions for plastic frames under combined M and N actions. The objectives of this paper are to present the algorithm for solving the characteristic problem of ultimate load limit analysis of elasto-plastic plane frames subjected to combined flexural moment and normal forces; numerical structural solutions based on the presented algorithm so as to demonstrate the feasibility of the technique as a practical tool in solving plane frame strength problem in professional practice.

ASSUMPTIONS

The presented algorithm of the method of analysis of plastic frames is applicable when the following assumptions are postulated:

i) The influence of shearing forces on section plastic moment are neglected.

ii) Plane sections under bending remain plane after deformation

iii) Material is ideally elastic-plastic, such that, strain hardening and residual stresses of the material are neglected. The stress - strain relationship is idealized to consist of two straight lines:

$$\sigma = E\varepsilon \qquad (0 < \varepsilon < \varepsilon_y)$$

$$\sigma = \sigma_y \qquad (y < \varepsilon < \infty)$$

iv) The properties of the element section in compression are the same as in tension or compression

v) The deformations are assumed to be sufficiently small so that equilibrium conditions can be formulated for the undeformed structure. Thus no effect of exial deformation on flexural stiffness and of flexural deformations on axial stiffness are considered.

vi) Buckling and local instability of the structure will not occur prior to the attainment of the ultimate load.

vii) Applied loads are static and monotonically increase up to structure failure.

viii) Plastic hinges are not modelled, they are assumed to be concentrated at critical sections, j. No allowance is made for the spread of yielding.

ALGORITHM FOR (M+N) METHOD OF SIMPLE ELASTO-PLASTIC LIMIT ANALYSIS OF PLANE FRAMES

Introduction

A set of unit forces is applied and their magnitudes are increased monotonously by increments. The magnitude of each unit load increment is just sufficient for a plastic hinge to form at the most stressed section of the structure. For each load increment, an elastic analysis based on direct Stiffness Method (DSM) is performed and the member end actions i.e. axial forces, bending moments (BMs) and displacements are computed. The BMs are then used to determine a minimum unit load multiplier Δ_i which causes the adjusted section plastic moment M_{pcj} to be reached at any section j. When M_{pcj} is attained at section j, a plastic hinge is inserted there and the global structure stiffness matrix $[K_g]$ changed accordingly to reflect the insertion of the plastic hinge. The load increment is continued until collapse mechanism is reached. This is identified by very large deflection of any node.

The Algorithm of (M+N) method of plastic analysis

The algorithm of the (M+N) technique is enlisted in the following stages.

1. Assemble structure geometrical and material properties arrays. This will automatically define the nodal displacement vector [q].
2. Assemble the global effective nodal load vector, {P}. This is equivalent to external nodal load vector since element loads are not allowed for. That is plastic hinges form at nodes only.

3. Establish a (3 x 3) element stiffness matrix, $[K_j]$ due to the flexural and axial internal loading, in local coordinates. There are four types of element stiffness matrices depending on element boundary conditions. In general notation:

Where K_{22}, K_{23}, K_{32} and K_{33} depends on the boundary conditions of element j.

a) $K_{22} = K_{33} = 4.0; K_{32} = K_{23} = 2.0$ when both node I and J are fixed

b) $K_{22} = K_{23} = K_{32} = 0.0; K_{23} = 3.0$ when a real hinge exist at node I and node J is fixed.

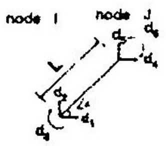
c) $K_{33} = K_{23} = K_{32} = 0.0; K_{22} = 3.0$ when a real hinge exist at node J and node I is fixed.

d) $K_{33} = K_{23} = K_{32} = K_{22} = 0.0$ when both node I and J are hinged.

4. Set up a (3 x 6) element displacement transformation matrix $[L_i]$. There are four types of transformation matrices depending on plastic conditions of the nodes of the element j.

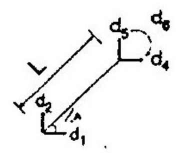
a) Element without any plastic hinges

$$[L_j] = \begin{bmatrix} -CosA & -SinA & 0.0 & CosA & SinA & 0.0 \\ \frac{SinA}{L} & \frac{CosA}{L} & 1.0 & \frac{-SinA}{L} & \frac{CosA}{L} & 0.0 \\ \frac{Sin\,A}{L} & \frac{-CosA}{L} & 0.0 & \frac{-Sin\,A}{L} & \frac{CosA}{L} & 1.0 \end{bmatrix}$$



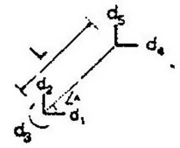
(b) Element with plastic hinge at node I only

$$[L_j] = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ \frac{-Sin\,A}{2L} & \frac{CosA}{2L} & 0.0 & \frac{Sin\,A}{2L} & \frac{-CosA}{2L} & \frac{-1}{2} \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$



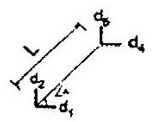
(c) Element with plastic hinge at node J only

$$[L_j] = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ \frac{-Sin\,A}{2L} & \frac{CosA}{2L} & \frac{-1}{2} & \frac{Sin\,A}{2L} & \frac{-CosA}{2L} & 0.0 \end{bmatrix}$$



(d) Element with plastic hinges at nodes I and J

$$[L_j] = \begin{bmatrix} -CosA & -Sin\,A & CosA & Sin\,A & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$



5. Form a (3x6) element stiffness matrix, $[K_j]' = [K_j]\,[L_j]$, in global coordinate system for elasto-plastic element, j.

6. Form a (3x6) element stiffness matrix, $[K_j]^{**} = [K_j]^{*}\,[L_j]$, in global coordinate system for the flexible elastic element, j. $[L_j]$, is the displacement transformation matrix of the flexible element assuming that the nodes of the element are hinged $.[K_j]^{*} = 10^{-8}\,[K_j]$ is the stiffness matrix of the flexible element placed parallel with the elasto-plastic element. The stiffness $[K_j]^{*}$ guarantees that singularity of the global stiffness matrix $[K_g]$ (obtained in stage 10 of this algorithm) does not occur while the system of nodal equilibrium equations are being solved.

7. Transform the (3x6) global element stiffness matrices $[K_j]'$ and $[K_j]^{**}$ to (6x6) matrices and sum up the two matrices.

$$[K_j]'' = [L_j]^T[K_j]^I + [L_j]^T[K_j]^{**}$$

8. Form global structure stiffness matrix as contributed by the elasto-plastic and flexible elastic elements

$$[K_g] = \sum_{j=1}^{n}[K_j]''$$

9. From global support node translational & rotational stiffness $[K_s]$.

10. Sum $[K_g]'$ and $[K_s]$ to form global structure stiffness matrix $[K_g]$

$$[K_g]'' = [K_g]' + [K_s]$$

11. Solve the system of nodal equilibrium equation $[K_g]\,\{q\} = \{P\}$ to obtain global nodal displacement vector, [q].

12. Extract from global nodal displacement vector, [q], the element end displacement vector, $[D_j\}'$.

13. Transform $\{D_j\}'$ to element node displacement vector, $\{D_j]$, in local coordinate axis system for:
    i) the elasto-plastic element $\{D_{1j}\} = [L_j]\,[D_j]'$ with $[L_j]$ as specified in stage 4 of this (M+N) algorithm
    ii) the flexible elastic element $\{D_{2j}\} = [L_j]\,\{D_j\}'$ with $[L_j]$ as specified in stage 6 of this (M+N) algorithm.

14. Establish the element nodal plastic deformation in local axis.

$$\{_{pl}D_j\} = \{D_{2j}\} - \{D_{1j}\}$$

15. Calculate element end moments ΔM_1^i and ΔM_2^i and axial force ΔN_l^i in local coordinate system due to unit loads.

$$\{S_j^i\} = \{\Delta M_1^i, \Delta M_2^i, \Delta N_l^i\}^T = [K_j]\{D_{1j}\};$$

Where i denotes ith cycle of analysis and subscript 1 and 2 being the two nodes of element I.

16. Extract element end moments from the array $\{S_j^i\}$ to give an array $\{S_{j(m)}\}$ which is composed of end moment of element j.

$$\{S_{j(m)}\} = \{\Delta M_1^i, \Delta M_2^i\}^T$$

17. Compute plastic work done in the respective plastic hinges(s), $[W_{pl}]$.

$$[W_{pl}] = \{S_{j(m)}\}\{_{pl}D_j\}^r;$$

Where $[_{pl}D_j]^r$ are plastic rotations extracted from $[_{pl}D_j]$

18. Check for closed plastic hinges of elements

i) If $[W_{pl}] \geq 0.0$ plastic hinge does not close

ii) If $[W_{pl}] < 0.0$ plastic hinge closes.

19. If plastic hinge closes, initialize the code for the closed hinge and repeat the process from first stage of the (M+N) algorithm.

20. Determine the minimum unit load multiplier for plastic hinge to form. This step involves five iterations by repeating steps 1 to 5 five times.

Step (1) Sort for minimum unit load multiplier Δ_i which will cause formation of plastic hinge(s) at ap propriate node(s).

$$M_j^{(i-1)} + \Delta M_j^i \Delta\lambda_i = M_{pcj}$$

$$\Delta\lambda_i = \frac{M_{pcj} - M_j^{(i-1)}}{\Delta M_j^i}$$

Where $M_j^{(i-1)}$ is the moment at node j of element l obtained from previous (i - 1)th cycle of analysis. If is the first cycle of elastic analysis them $M_j^{(i-1)} = M_j^o = 0$; ΔM_j^i is the moment at node j of element l due to unit load as computed in the ith cycle of analysis. It can either be ΔM_1^i or ΔM_2^i depending on the section j under consideration. It is computed in stage 15 of (M+N) algorithm; $\Delta\lambda_i$ is the minimum unit load multiplier being sort for and M_{pcj} is the adjusted plastic moment at section j after consideration the axial forces effects.

Step (2) Estimate the magnitude of axial force in element l in the ith cycles of analysis, $N_l i$

$N_l^i = (\lambda_c^{i-1} + \Delta\lambda_i)\Delta N_l^i$; Where $\Delta N_l i$ is the axial force in element l due to unit loads as computed in the ith cycle of analysis. This is obtained in 15th stage of (M+N) algorithm and,

$$\lambda_c^{(i-1)} = \sum_{k=1}^{(i-1)} \Delta\lambda_k$$

The cumulative load factor from first cycle of analysis to (i-1)th cycle of analysis.

Step (3) Compute the ratio, RAF is given by and is equivalent to n_j

$$RAF = \left\| \left[\frac{N_l^i}{N_{pl}} \right] \right\|$$

where N_{pl} is the axial yield strength of element l

Step (4) Compute the value of adjusted plastic moment, M_{pcj}, at section j. This is obtained from the following expressions.

i) $M_{pcj} = M_{pj}$ if $RAF \leq 0.15$

ii) $M_{pcj} = 1.18\,(1\text{-}RAF)\,M_{pj}$ if $RAF > 0.15$

Step (5) Repeat the process starting from step (1) by substituting M_{pcj} with improved section plas tic moment M_{pcj} computed in step (4) and obtain improved value of $\Delta\lambda_i$

Step (6) Repeat the procedures in steps (1) to (5) five times. The five iterations were found sufficient to give a reliable unit minimum load multiplier, $\Delta\lambda_i$ in ith cycle of analysis.

21. Introduce plastic hinge(s) at node(s) giving minimum unit load multiplier, $\Delta\lambda_i$ by setting new hinge codes.

22. Compute commutate load factor $\lambda_c^i = \lambda_c^{i-1} + \Delta\lambda_i$ as at ith cycle of analysis.

23. Compute updated element elastic nodal displacements, $\{D_j i\}$

$$\{D_j^i\} = \{D_j^{i-1}\} + \Delta\lambda_i\{D_{1j}\};$$

Where $\{D_j^{i-1}\}$ is the nodal horizontal & vertical deflection and rotation of node j in (i-1)th cycle of analysis; Δ_i is the minimum unit load multiplier in the ith cycle of analysis obtained in stage 20 of this algorithm, $[D_{lj}]$ is the respective local nodal displacement of elasto-plastic element due to unit load. It is obtained in 13th stage of this algorithm.

24. Check for collapse mechanism and output results if mechanism has formed.

25. Compute updated element forces and nodal bending moments at the end of ith cycle of analysis:

$\{N_l^i\} = \{N_l^{(i-1)}\} + \Delta\lambda_i\{\Delta N_{li}\}$ for axial force in element l

$\{M_j^i\} = \{M_j^{(i-1)}\} + \Delta\lambda_i\{\Delta M_{ji}\}$, for moment at node j of element l.

26. Repeat the algorithm from 1st stage for the next ith cycle of (M+N) first order-elasto-plastic limit analysis.

COMPUTATIONAL TECHNIQUE OF (M+N) ALGORITHM

Basing on the above algorithm a program first order Elasto-Plastic Limited Analysis of structural plane Frames, (EPLAF) [13] have been developed. EPLAF provides a compute aided structural analysis procedure for determining the ultimate strength capacity of steel plane frames subjected to combined flexural moment and axial thrust. The program computes the node displacements, member forces, support reactions, plastic hinge rotations and collapse loads for plane frames of any arbitrary shape. The program also yields the location, sequence of formation of plastic hinges and the load factor at which the hinges form.

The program is written in standard FORTRAN-77 code for compilation by Microsoft FORTRAN 4.2 Optimizing compiler. The listing of the program is given in Plate AP1.4 of Appendix 1 of reference (13).

APPLICATIONS

A four story frame with the geometry and loading in Fig. 3 and member properties in Table 1 was analysed by using (M+N) Plastic analysis technic whose algorithm has been presented above. Numerical results thus obtained are summarized in Figs. 4 to 6. Also included in the figures are numerical solutions to analysis of the same frame by using sudhakar's (1986) Scheme which neglects effects of axial force in the analysis.

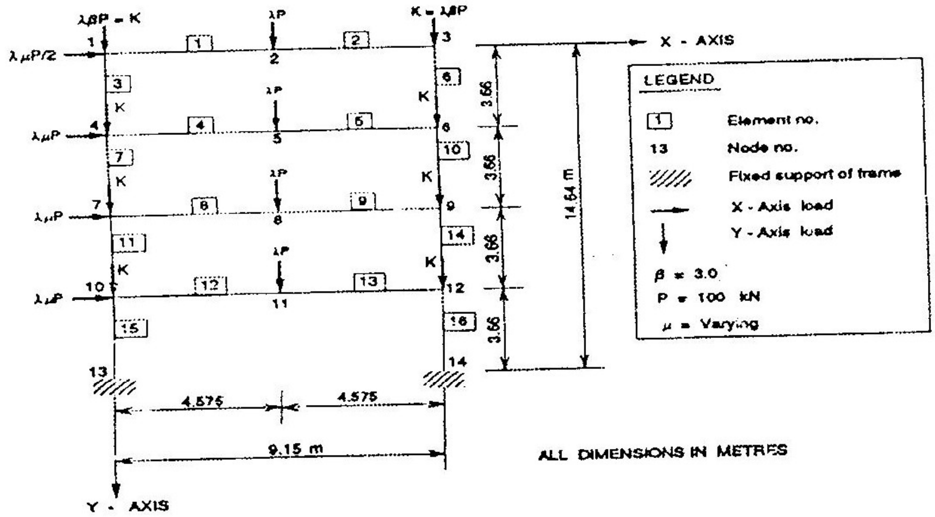


Fig. 3 **Geometry of Frame-A and general loading configuration acting on it.**

Table 1 Structural properties of Frame-A

Element group	Elements in the group	Section Plastic moment M_{pl} (kNm)	Section Normal yield force, N_{pl}, (kN)
UB	1,2,4,5,8,9,12 and 13	282.020	1795.96
UC1	3,6,7 and 10	289.808	2690.40
UC2	11,14,15 and 16	439.196	3957.72

Ultimate Load Factor

The load factor at incipient structure collapse predicted by the presented algorithm was 1.854 while that predicted by Sudhakar's procedure was 2.128 giving a discrepancy of 12.8% when the lateral load multiplier is 0.24. Thus the new algorithm predicts reduced ultimate load carrying capacity of the frame.

Load deformation behaviour of the frame

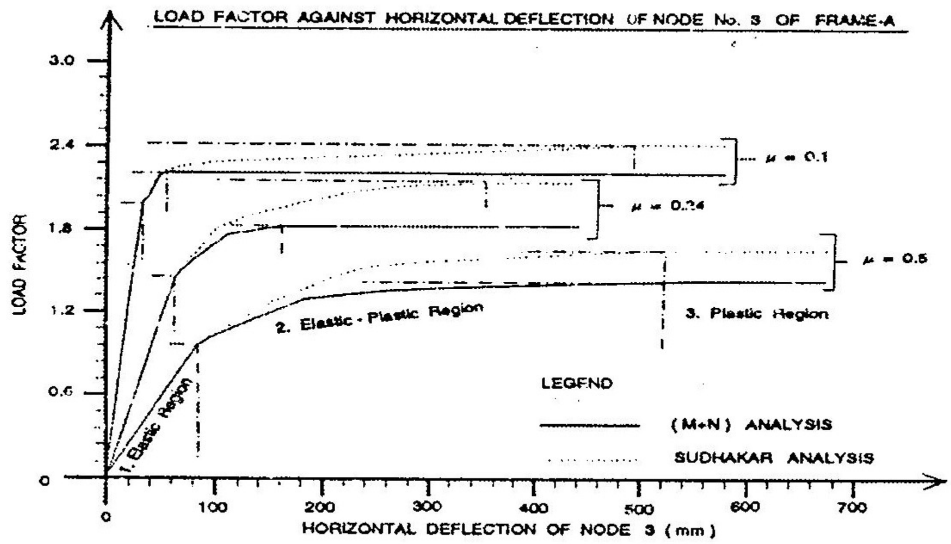


Fig. 4 **Load factor-horizontal deflection curves for node no. 3 of Frame-A when analysed by (M+N) & Sudhakar's methods of analysis.**

The full range load deflection curves for the Frame-A in Fig. 3 is shown in Fig. 4 for both types of analyses on the same figure, curves for varying values of lateral loads are shown. It is observed that from the moment the frame becomes elastic-plastic the load required to yield any deflection is less in (M+N) method than in Sudhakar's method. In essence inclusion of axial force in plastic analysis leads to increase in stiffness degradation of the frame.

Plastic behaviour of the frame

Fig. 5(a) and 5(b) summarises the plastic behaviour of Frame-A as predicted by (M+N) and Sudhakar's methods of analysis. It is evident from the figures that the location & sequence of formation of plastic hinges and the corresponding load factors at which they formed, tallied only in the first two cycles of both schemes of analysis. In subsequent cycles of analysis, the sequence of formation of plastic hinges and the location at which they formed differed. At onetime the number of plastic hinges which formed were twelve in the new enhanced scheme and ten in Sudhakar's scheme. The two extra plastic hinges formed at (10)6 (to be read as node no. 6 of element 10) and (6)6. (M+N) analysis predicted combined joint and frame sway mechanism while Sudhakar's analysis predicted frame sway mechanism as collapse modes. Both the collapse modes were partial collapse mechanisms.

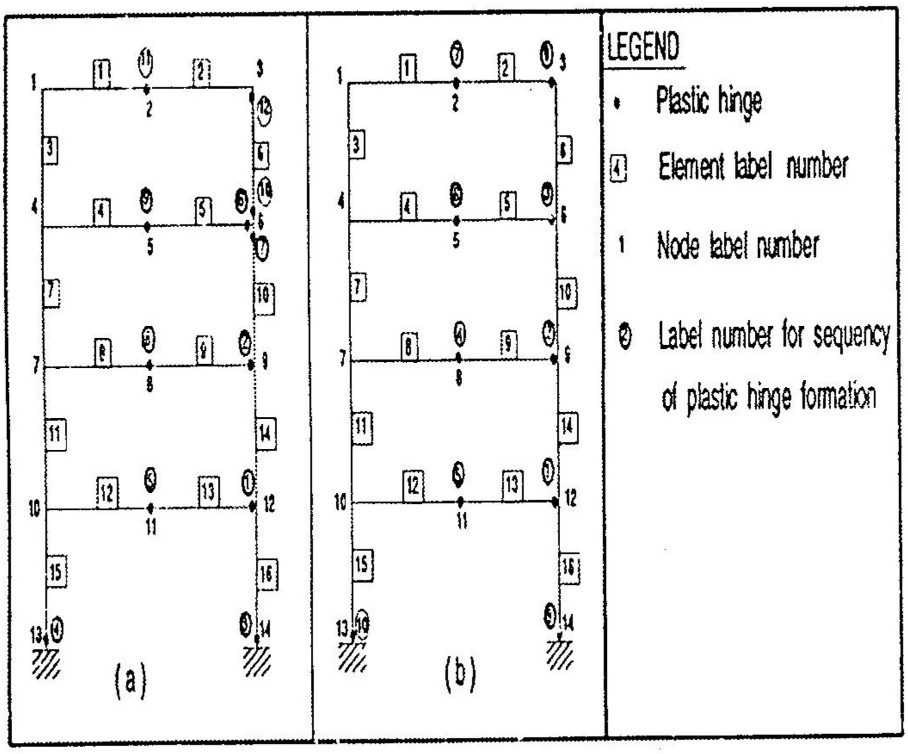


Fig. 5 **Location and sequence of formation of plastic hinges in Frame-A as analysed by:(a) (M+N) technique (b) Sudhakar's technique.**

Bending Moment Distribution in the frame

Fig 6 show the BMDs in Frame-A mechanism as computed by (M+N) and Sudhakar's analyses. It is observed that incorporating axial force effects in first order elasto-plastic limit analysis adversely affects distribution of bending moments in the collapse mechanism of the frame. The maximum percentage differences in nodal BM_s computed by the two techniques were 88.3% for UB, 28.2% for UC 1 and - 143.1% for UC2.

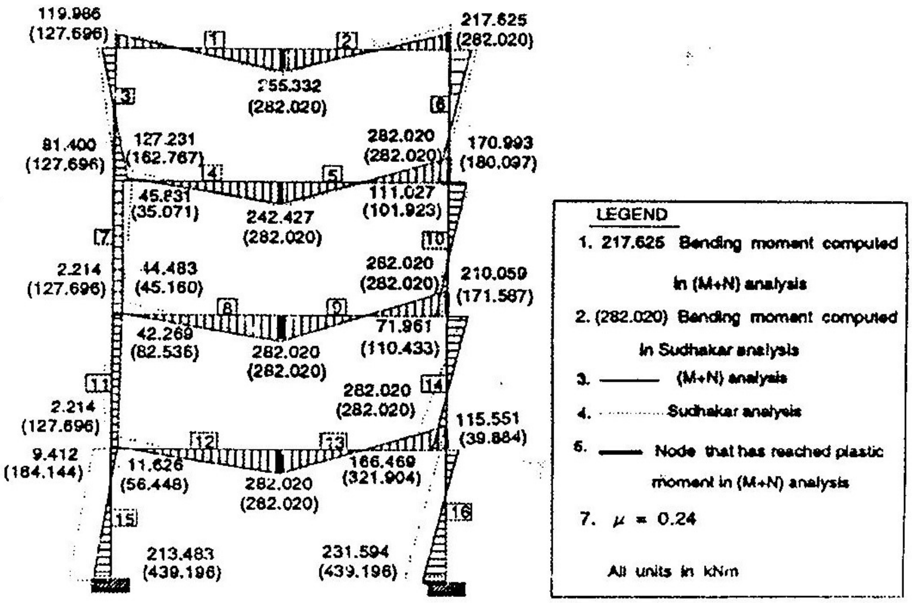


Fig. 6 Bending moment distribution in frame-A mecahnism

CONCLUSIONS

i Simple elastic-plastic limit analysis of steel structural plane frames has been enhanced to include the influence of axial forces.

ii EPLAF program, derived from ULARC, provides an efficient method for full range analysis of steel structural plane frames subjected to static monotonously increasing loads.

iii The consideration of normal forces in simple elasto-plastic analysis of structural plane frame system; Reduces the load carrying capacity of the frame; Increases the rate of stiffness degradation of the frame; Adversely affects distribution of bending moments in the collapse mechanisms of the frames; Changes the type of collapse mechanism that is formed with the most common change being from beam mechanism to either sway or combined beam and sway mechanism modes; Alters the number of plastic hinges that are formed in the frame provided that the type of mechanism formed is a partial collapse mechanism; Changes the sequence of formation of plastic hinges in the structure; Alters the locations of some of the critical sections, in most cases the change was from a node of a beam element to a node of a column element. The above plastic behaviour conclusions depend on the magnitude and configurations of the loads subjected to the structure. the conclusion are valid when the applied load cause the critical sections of the frame to transmit normal forces which are greater than 15% of the respective element plastic axial force.

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