COMPUTER MODELLING OF STEADY STATE FLOW OF FLUIDS IN PIPES

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ABSTRACT

Steady state analysis is applied to fluid flow systems under steady state conditions or for establishing the initiat flow conditions, when performing transient and unsteady flow analysis. Four different models, all of which assume viscous flow, were developed. The models are an incompressible flow model and three compressible flow models based on isothermal, adiabatic and non-isothermal non-adiabatic flow assumptions. Three numerical methods of solution were used, only for incompressible adiabatic flow assumption. The methods are first-order backward, first-order forward and second-order finite difference methods. For the rest of the flow assumptions only the first-order backward finite difference method was used. The QUANT software and also the general equation of state were used to calculate the thermodynamic and transport properties of fluids. The non-isothermal non-adiabatic model, using the first-order backward numerical method of solution, was compared with predictions made for the proposed Songo Songo Dar es Salaam Natural Gas Pipeline. A significant discrepancy was observed between the two results. The comparison made between the three numerical methods used in conjunction with the adiabatic flow model revealed that for long pipelines and flow velocities which are significantly greater than zero, there could be significant variations in the results from the three numerical methods. The comparison which was made between the first- and second-order methods reveals that the second-order method produces results which are significantly more accurate than the first-order methods.

INTRODUCTION

When a steady state pattern of flow in a pipe is disturbed, a pressure wave is formed. The pressure wave travels away from the source of disturbance. The possible causes of the perturbance are the changes in boundary conditions such as valve closing and opening, pressure change in supply or receiving ends, pump start up or shut down, change in inflow or outflow of the system etc. In steady state flow, there is no change in flow condition such as pressure, velocity and mass flow rate at a point with time. In practical flow problems, the temporal mean conditions are used when referring to steady state flow. For example, strictly speaking, turbulent flows are always unsteady since the conditions at a point are changing continuously. However, by considering temporal mean values over a short period of time, turbulent flow could be considered as steady if the temporal mean condition do not change with time. Before the intoduction of computers, all calculations of pressure and flow in gas transmission line were based on steady state flow.

Steady state analysis is a very important tool in Computational Fluid Dynamics. It is used to obtain the initial flow conditions prior to performing transient or unsteady flow modelling. The initial condition is the steady-state condition in the pipe prior to the initiation of unsteady flow The initial conditions are also specified at the boundaries at the initial time.

THE BASIC EQUATIONS FOR STEADY STATE FLOW

The basic equations for steady state flow analysis were derived from the basic equations for unsteady flow [1]. All the partial derivatives with respect to t were set to zero, thus resulting in a set of ordinary differential equations with respect to x. Assuming that the cross-sectional area of the pipe is constant, the three ordinary differential equations are as follows:

Continuity equation

$$u\frac{d\rho}{dx}+\rho\frac{du}{dx}=0 \tag{1}$$

Momentum equation

$$\frac{dp}{dx}+\rho u\frac{du}{dx}+\left(\frac{\omega}{\rho A}+g\sin\theta\right)\rho=0 \tag{2}$$

Energy equation

$$\frac{dp}{dx} + \rho \frac{a^2}{u}\frac{du}{dx} - \frac{1}{A}(\delta_s - 1)\left(\frac{\Omega}{u} + \omega\right) = 0 \qquad (3)$$

The general equation of state for a real gas is as follows:

$$P = Z\rho RT \qquad (4)$$

The speed of sound for a perfect gas is calculated using the following equation:

$$a = \sqrt{p\frac{K}{\rho}} = \sqrt{KZRT} \qquad (5)$$

and for a real gas

$$a = \sqrt{\frac{p\gamma_s}{\rho}} = \sqrt{\gamma_s}ZRT \qquad (6)$$

The QUANT software for thermodynamic and transport properties of fluids was used. Also the general equation of state [equation (4)] was used in cases other than the non-isothermal non-adiabatic compressible flow Given the fluid properties at one grid point, the equations were solved simultaneously to obtain the properties at the next grid point.

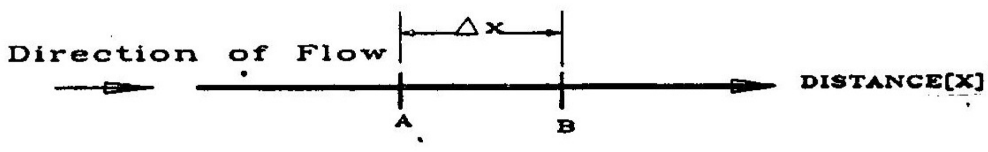

Fig. 1 Grid Mesh for Steady State Flow Modelling

After calculating p, u and ρ; the temperature of the fluid was calculated using either the QUANT software or the general equation of state [equation (4)]. The speed of sound was calculated using either equation (5) or (6), depending on the flow assumptions made.

INCOMPRESSIBLE FLOW

The incompressible flow assumption means that the rate of change of fluid density in the axial direction $(d\rho/dx)$ is zero. According to the continuity equation [equation (1)] this implies that the rate of change of flow velocity in the axial direction (du/dx) is also zero. In one variation, it was assumed that temperature is constant i.e. isothermal flow, while in another variation an adiabatic flow was assumed. In both assumptions, numerical analysis was performed using a finite-difference method in which the fluid properties at the previous grid point were used to calculate the new fluid properties at the new grid point. This numerical method is referred to as the backward finite-difference method, and is of the first-order of accuracy. Using this method, equation (2) and referring to Fig. 1, the pressure drop between the two points is given by the following equation:

$$\Delta p = -\left(\frac{\omega}{\rho A} + g\sin\theta\right)\rho\Delta x \qquad (7)$$

ρ and u are constant for both the isothermal and adiabatic models andT is also constant for the isothermal model. The temperature drop in the adiabatic model was obtained by differentiating the general equation of state [equation (4)], which in finite differences becomes the following equation:

$$\Delta T = \frac{\Delta p}{ZR\rho} \qquad (8)$$

ISOTHERMAL COMPRESSIBLE FLOW

The continuity equation [equation (1)], the momentum equation [equation (2)] and the differential form of the general equation of state [equation(4)] with respect to ρ were solved simultaneously. Referring to Fig. 1, the finite-difference equations based on the fluid properties at the previous grid point are as follows:

$$\Delta p = -\frac{\left(\dfrac{\omega}{\rho A} + g\sin\theta\right)}{\left(1 - \dfrac{u^2}{ZRT}\right)}\rho\Delta x \qquad (9)$$

$$\Delta u = -\frac{u}{\rho ZRT}\Delta p \qquad (10)$$

$$\Delta\rho = -\frac{\rho}{u}\Delta u \tag{11}$$

and T is constant.

ADIABATIC COMPRESSIBLE FLOW

The continuity equation [equation (1)], the momentum equation [equation (2)] and the equation of state for adiabatic flow were used. The equation of state is defined as follows:

$$\frac{p}{\rho^n} = k \tag{12}$$

where n is the polytropic coefficient of the gas and k is a constant. Alternatively, equation (4) could be used. Expressing equation (12) in natural logarithms, assuming n is constant and differentiating the following equation was obtained:

$$d\rho = \frac{\rho}{n}\frac{dp}{p} \tag{13}$$

Similarly from equation (4) and assuming that Z is constant.

$$dT = \left(\frac{dp}{p} - \frac{d\rho}{\rho}\right)T \tag{14}$$

Three different numerical methods were used to solve the four differential equations (1), (2), (13) and (14). The three methods differ in the properties of the fluid used to calculate the dependent variables at the new grid point i.e. point B in Fig. 1. In the first method, the fluid properties used were those at the previous grid point (point A). This method is referred to as first-order backward difference method. In the second method, the properties used were those at the new grid point (point B) and the method is called first-order forward difference method. The third method, is referred to as a second-order difference method and uses the average of the properties at the previous and new grid points i.e. points A and B respectively in Fig. 1. The full derivation of the three methods was presented[2].

(a) First-order Backward Difference Method

In this method, the dependent properties of the gas p, u, ρ and T; were calculated based on the properties of the gas at the previous grid point.

The four differential equations (1), (2), (13) and (14) were used to derive the finite-difference equations for this method. The resulting finite-difference equations are as follows:

$$\Delta u = -\frac{u}{n}\frac{\Delta p}{p} \tag{15}$$

$$\Delta p = \frac{\left(\dfrac{\omega}{\rho A} + g \sin\theta\right)}{\left(\dfrac{\rho u^2}{np} - 1\right)} \rho \Delta x \tag{16}$$

$$\Delta \rho = -\rho \frac{\Delta u}{u} = \frac{\rho}{n}\frac{\Delta p}{p} \tag{17}$$

$$\Delta T = \left(\frac{\Delta p}{p} - \frac{\Delta \rho}{\rho}\right) T \tag{18}$$

The other thermodynamic and transport properties of the fluid were also calculated, based on the state of the fluid at the previous grid point.

(b) First-order Forward Difference Method

Calculation of the four dependent gas properties p, u, ρ andT at the new grid point was effected using values of the same properties at the new grid point. But the other thermodynamic properties and also the transport properties used were based on the previous grid point. Equation (2) was expressed in finite differences. Equations (15) and (17) were used to calculate Δu and Δρ respectively. The resulting values of Δu and Δρ were substituted into the finite-difference form of equation (2) to obtain a fourth order polynomial function in Δp. The polynomial function is as follows:

$$[(n+1)^2 (n-1)]\underline{(\Delta p)}^4 + [2n(n+1)(n-1)p + n(n+1)^2 p - n^2 pu^2 + n(n+1)^2 p\, l\, \Delta x]\underline{(\Delta p)}^3 + [2n^2 (n+1)p^2 + n^2 (n-1) p^2 - 2n^2 ppu^2 + n(n+1)(3n+1)pp\, l\, \Delta x]\underline{(\Delta p)}^2 + [n^3p^3 - n^2p^2 pu^2 + n^2(3n+2)p^2 pl\, \Delta x]\underline{(\Delta p)} + n^3 p^3 p\, l\, \Delta x = 0 \tag{19}$$

where

$$l = \frac{\omega}{\rho A} + g \sin \theta \qquad (20)$$

The polynomial function was solved for Δp, which was then used to calculate Δρ and Δu. Equation (19) was solved using a numerical recipe, which is a modified version of recipe XZRHQR [3] and [4]. The fluid temperature was calculated using equation (14), but expressed as forward differences.

(c) Second-order Finite-Difference Method

In this method, the fluid properties were based on the averages between the properties at the previous and new grid points i.e. points A and B respectively, in Fig. 1. The procedure used for solution of the basic equations is very similarly to the one used for the first-order forward method. This also resulted in a fourth-order polynomial function in Δp, which was solved by the same numerical recipe XZRHQR[3] and [4]. The fourth-order polynomial function is as follows:

$$(n-1)(n+1)^2 (\underline{\Delta p})^4 + [2np(n+1)^2 + 4np(n+1)(n-1) - 2n^2 u^2 p + n(n+1)^2 p\, l\, \Delta x](\underline{\Delta p})^3 + [4n^2 p^2 (3n+1) - 8n^2 u^2 pp + 2n(n+1)(3n+1)pp\, l\, \Delta x](\underline{\Delta p})^2 + [8n^3 p^3 - 8n^2 p^2 u^2 p + 4n^2 p^2 (3n+2)\, p\, l\, \Delta x](\underline{\Delta p}) + 8n^3 p^3 pl\, \Delta x = 0 \qquad (21)$$

Also in this case, the fluid temperature was calculated using equation (14), but expressed as second-order differences.

NON-ISOTHERMAL NON-ADIABATIC COMPRESSIBLE FLOW

All the three basic equations for steady state flow analysis, equations (1), (2) and (3), were used in the non-isothermal non-adiabatic compressible flow model. The QUANT software was used to calculate the thermodynamic and transport properties of the fluid. The fluid properties at the previous grid point were calculated using the QUANT software. The values of Ω and ω were calculated using the properties determined above. Equations (1), (2) and (3) were solved simultaneously for ϕ, du and dp and the resulting equations were transformed into finite-diference equations to obtain Δp, Δu and Δρ. In all the calculations, the fluid properties were based on the properties at the previous grid point. The values of p, u and ρ at the new grid point were calculated. The fluid temperature at the new grid point was calculated using the QUANTsoftware. The procedure was repeated for the subsequent grid points.

Equations (2) and (3) were solved simultaneously for Δu and the resulting finite-difference equation is as follows:

$$\Delta u = -\frac{\left[(\delta_s - -1)\left(\frac{\Omega + \omega u}{A}\right)\frac{1}{u} + \left(\frac{\omega}{\rho A} + g\sin\theta\right)\rho\right]}{\rho u\left[1 - \left(\frac{a}{u}\right)^2\right]}\Delta x \qquad (22)$$

Equations (2) and (1) were used to calculate Δp and Δρ respectively.

The C programming language was used to develop computer codes for all the different flow cases which have been described. The non-isothermal non-adiabatic flow model was used to establish the initial flow conditions, when modelling unsteady and transient flow following linebreak in high-pressure gas pipelines [2]. The steady state model assumes a fully developed flow, with the fluid properties being known at the initial grid point. After prompting for system data and in the case of variable grid size generating the distance grid, steady state analysis was performed for all the distance grid points in both the upstream and downstream sections of the pipe. Two separate programmes were written, one for each of the pipe sections. The programme for the downstream section depends on data calculated by the programme for the upstream section. This makes it necessary for the former programme to be run in succession to the latter programme.

TESTING OF THE COMPUTER MODEL

A comparison was made between the three numerical methods used in conjunction with the adiabatic flow model with regard to their accuracy and simplicity. The data used were for relatively short pipelines and zero initial flow velocity.

The computer model for non-isothermal non-adiabatic flow was tested with preliminary design data for the Songo Songo to Dar es Salaam natural gas pipeline [5]. Three demand scenarios namely low, medium and high were considered but only the high demand scenario was used. The design based conditions were a pressure of 101kPa (14.65psig), a temperature of 15°C and a specific gravity of 0.60. The maximum allowable operating pressures were set at 7.93MPa for the low demand scenario and 9.65MPa for both the medium and high demand scenarios. The maximum allowable gas temperature was 49°C, while the maximum design gas temperature was set at 38°C.

Pipe sizes and flow capacity of the pipeline were determined[5] using the Panhandle formula, which considers gas pressure, temperature and velocity Steel, plastic and aluminium were all considered as possible materials for construction of the pipeline. However, it was later decided that only a steel pipe would be able to carry the volumes and pressures required for the transmission line. The gas velocities in the pipeline, for each scenario, pipe size and pressure were estimated as shown inTable 1. The pipe sizes were designed to allow fluctuations which may occur between the peak day demand and the average flow, and the expected velocities are well below the target velocity for the pipeline design. In this particular case flow velocities of 7.6m/s and 10.2m/s were given as target and maximum velocities respectively. Pipe sizes were chosen for the three scenarios, based on the initial pressure of 4.80MPa available at Kilwa Kivinje and a minimum terminal pressure of 2.07MPa (300psig) at the city gate station in Dar es Salaam. For both the medium and high demand scenarios, compression was to be added at Kilwa Kivinje.

Table 1: Estimated gas velocities, pipe sizes and presure

SCENARIO	VELOCITY [m/s]	NOMINAL DIAMETER [mm]	PRESSURE [MPa]
LOW	3.3	254	-
MEDIUM	3.9	254	9.65
HIGH	4.3	305	9.65

Source: Hardy BBT Ltd. (1989)

Predicted line pressure profiles for the high demand scenario are presented in Fig. 3. For each scenario there are three curves representing maximum sustainable flow consistent with a discharge pressure of 2.07MPa (300psig), design flow rate similar to maximum average daily demands for the three scenarios and flow rate of 70% of the above design flow rate. The distribution system is comparatively small so that while it serves to smooth out local variations in demand, it can not be considered as a major component of storage. The major storage in this case is provided by on-line storage in the transmission pipeline.

Three block valve locations were proposed in the preliminary pipeline design. These would permit each section of the transmission line between the valves to be isolated and blown down during repair, maintenance or in an emergency situation. Operation of the block valves would be in the manual mode. Automatic linebreak control was not considered in the preliminary design. It was suggested that additional block valves could be added to the system during the final stage of the project depending on operational considerations. The three block valve locations are presented in Fig. 2.

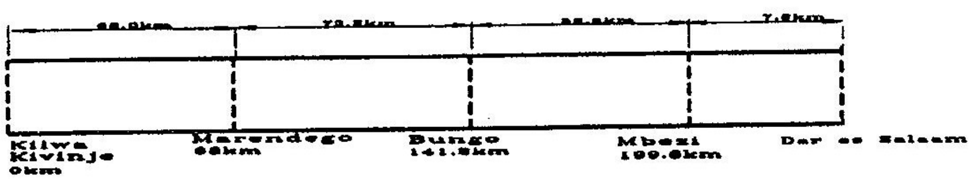

Fig. 2 Block Valve Locations

The original design of the pipeline, as presented by Hardy BBT Ltd.[5], was for the pipeline to pass through Kilwa Kivinje and also to be tied to the project for the fertilizer plant at Kilwa Masoko. Recent information, including a brochure published by the Ministry of Water, Energy and Minerals and two Canadian companies involved in the implementation of the project [6], indicate that a new pipeline route is being followed. According to the new route, the pipeline is from Songo Songo directly to Dar es Salaam, through Somanga Funga and not Kilwa Kivinje. The new pipeline route does not seem to be tied to any other project apart from the power station and other users in Dar es Salaam. Although it is expected that design details for the new route have been finalised, it was not possible to get them for this study. However, since the pipeline length does not differ much from the one in the proposed old route, and assuming that the gas demand scenarios in Dar es Salaam remain the same, the same data which was proposed for the old pipeline design was used.

A data file to be used by the CFD model was generated from the QUANT software. The initial pressure and mass flow rate of the gas at Kilwa Kivinje were 1410psi (9.65MPa) and 100.79MMCFD (25kg/s). The maximum operating temperature of the gas was 311K. The rest of the gas properties at the initial conditions at Kilwa Kivinje were calculated using the QUANT software. Since this model was used to generate the initial conditions for modelling of linebreak in the pipeline[2], a minimum grid spacing of $\Delta x = 10m$ and $\Delta t = 0.01s$ at the broken end and the variable grid model were used.

The Hardy BBT prediction data[5], which is presented in Fig. 3, were used for comparison with the data calculated using the non-isothermal non-adiabatic steady state analysis model and also the transient analysis, before the break, using the method of characteristics[1]. The prediction results obtained are included in Figs. 3. Transient analysis before the break was performed in order to establish the initial unsteady flow condition. In both the steady state and transient analyses, variable grid spacing programmes were used. The heat transfer model which was used is one which considers heat transfer from a buried pipe.

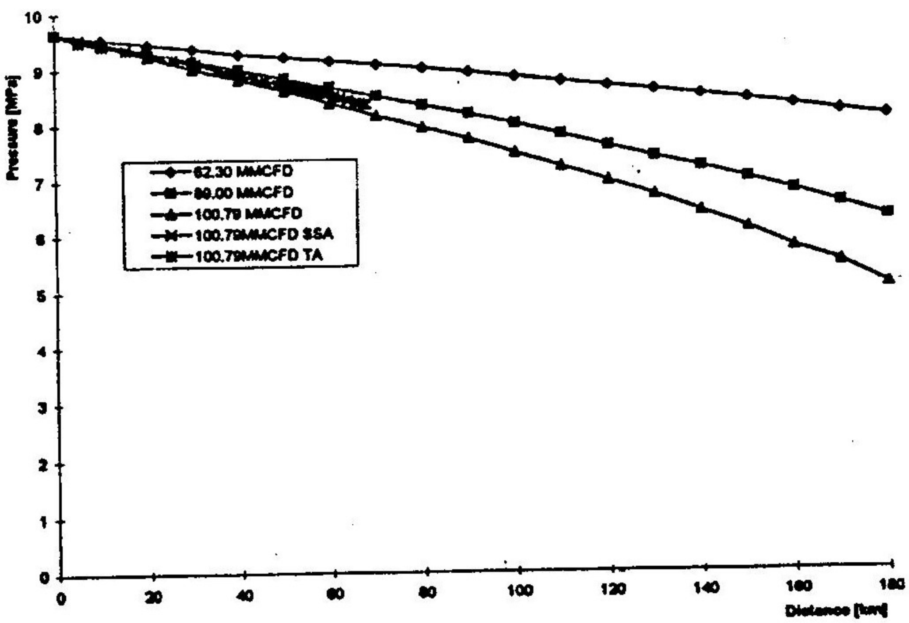

Fig. 3: Pressure profile in Songo Songo Dar es Salaam gs pipeline in comparison with model predictions

DISCUSSION OF SIMULATION RESULTS

It was generally found that there is not much difference in the results from the three methods. The first-order backward method was selected for use in all the steady state analysis programmes, especially because of its simplicity. Also since the steady state model was by itself a rough representation of the flow of gas in high-pressure pipelines, it was concluded that there was no justification for using a more complicated numerical method. However, for longer pipelines and flow velocities which are significantly greater than zero, variations in the results from the three numerical methods are expected. Simulation results for the pressure profile using the steady state analysis (SSA) and transient analysis (TA) models are presented in Fig. 3, together with the original data provided by Hardy BBT Ltd. [5].

The pressure profiles for steady state analysis and transient analysis are also presented in Fig. 4. In calculating the pressure profile using the steady state analysis model, problems were encountered when the pipe diameter of 0.305m, which was specified by Hardy BBTLtd. [5], was used. The use of this pipe diameter and gas flow rate of 25kg/s, which were given, resulted in a much bigger pressure drop in the pipeline. The pressure predicted at Marendego (68km from Kilwa Kivinje) was around 4MPa, which is less than a half of the pressure presented by Hardy BBT Ltd [5]. Transient analysis using the method of characteristics was performed, in order to confirm whether the problem was with the steady state analysis model or the Hardy BBT design data. When a boundary condition corresponding with the pressure presented in the Hardy BBT curves (8.2MPa) was used, negative velocities were obtained at the low pressure end. The negative velocities indicate that the pressure imposed at the boundary was higher than the actual pressure for the parameters given. The mass flow rate was reduced by a half in order to see if the pressure drop would decrease to the required. value, but the pressure drop remained higher When the pipe diameter was increased to 0.5m, the same pressure profile as that presented by Hardy BBT was obtained. Transient analysis was then performed and produced the same pressure profile as the steady state analysis. However after the transient analysis, the flow velocity was slightly higher than that obtained using the steady state analysis model, but within the limit specified by Hardy BBT. This result indicates that the pipe diameter of 0.305m which was specified in the preliminary design Hardy BBT Ltd.[5] was too small.

There was no output from the QUANT software for the ccoefficient of dynamic viscosity (μ) for all the range of pressure and temperature used. A constant value was used throughout the range of pressure and temperature encountered. The constant value was chosen such that it is in the middle of the range, therefore reducing the error in estimating μ.

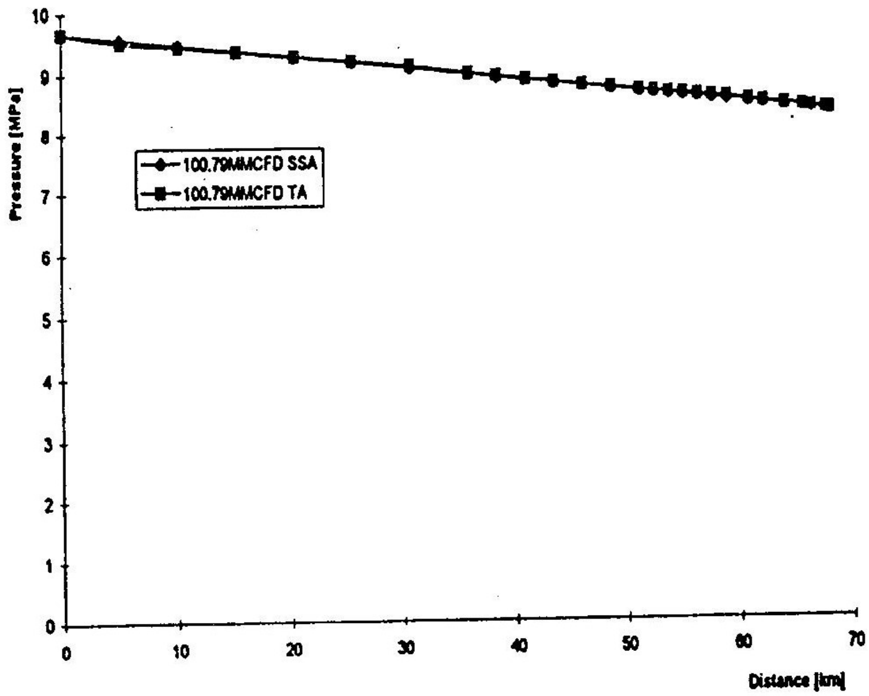

Fig. 4: Predicted pressure profile in Songo Songo Dar es Salaam pipeline

CONCLUSIONS

Four different models for calculating the steady-state conditions, were developed. The four models, all of which are viscous flow models are based on the assumptions of incompressible flow and compressible isothermal, adiabatic or non-isothermal non-adiabatic flow Three different numerical methods for solution of the theoretical models were used. The three numerical methods are a first-order backward method, a first-order forward method and a second-order method.The models were developed into pc based computer codes using the C programming language. The non-isothermal non-adiabatic model, using the first-order backward numerical method of solution, was compared with predictions made for the proposed Songo Songo Dar es Salaam Natural Gas Pipeline by Hardy BBT Ltd. [5]. There were significant variations between the steady-state analysis model predictions and predictions which were made for the Songo Songo Dar es Salaam pipeline by Hardy BBTLtd. [5]. This discrepancy is attributed to the small pipe diameter (0.305m) which was specified by Hardy BBTLtd. [5]. When the pipe diameter of 0.5m was used both results were in good agreement. The comparison made between the three numerical methods used in conjunction with the adiabatic flow model revealed that for longer pipelines and flow velocities which are significantly greater than zero, there could be significant variations in the results from the three numerical methods.

The comparison which was made between the first-order methods and the second-order method for solution of the steady-state flow equations reveals that the second-order method produces results which are significantly different and more accurate than the first-order methods. In this study the steady-state analysis results were used just as initial estimates, and were later improved by transient analysis. Therefore the first-order backward method was sufficient. In cases where more accurate steady-state analysis results are required, the second-order method is recommended.

NOMENCLATURE

A	= Cross-section area of pipe
a	= Wave speed
g	= Gravitational acceleration
h	= Specific enthalpy of gas
p	= Static pressure of gas
R	= Specific gas constant
t	= Time
u	= Flow velocity of gas
x	= Horizontal distance along the pipe
Z	= Compressibility factor of gas

Greek Symbols

γ_s	= Isentropic gamma coefficient
Δ	= Small change in the quantity
δ_s	= Isentropic delta coefficient
θ	= Angle of inclination of pipe to horizontal

μ	= Coefficient of dynamic viscosity
ρ	= Density of gas
Ω	= Heat flow into the pipe per unit length of pipe per unit time
ω	= Frictional force per unit length of pipe

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