PREDICTION OF GROUNDWATER LEVELS IN THE EWASO NYIRO BASIN USING THE FINITE ELEMENT METHOD

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ABSTRACT

The importance of predicting groundwater profiles when groundwater is exploited for water supply has been explained. A finite element code based on the two dimensional quasi harmonic equation was used to predict groundwater potentials in the Ewaso Nyiro catchment basin located in Central Kenya. The predicted potentials were compared to the actual potentials measured for some of the boreholes located in the basin. Basin on the results, it was found that the finite element method is a useful numerical tool in predicting groundwater potentials.

INTRODUCTION

The potential of groundwater as a resource for fresh water is well recognised [1]. Its exploitation is particularly attractive for drinking water and agricultural purposes since in most cases little or no treatment at all is required, a characteristic that is very appealing for community water supply in developing countries.

The harvesting of groundwater is principally through boreholes. In the absence of a piped supply system, as is the case in most developing countries, many boreholes have to be provided dispersed within the community.

The sinking of boreholes is expensive. Accordingly, tools that give guidance on the proper location and the likely depth to the groundwater are very useful and can lead to substantial cost savings. Such tools can also assist in proper management which is essential in order to avoid excessive exploitation and overpumping that often lead to salination and land subsidence [2].

In this paper, an existing finite element code is applied to a typical catchment area in Kenya in order to assess its suitability and versatility in predicting groundwater potentials. The code, developed by researchers at the Free University of Brussels, Belgium [3] is based on a model for groundwater flow under saturated conditions.

REVIEW OF THE BASIC FLOW EQUATIONS

The flow equations are developed through a macroscopic continuum approach [3]. Any volume considered is large enough and is called a Reference Elemental Volume (REV). Variables are considered on a macroscopic scale and hence laws of continuum mechanics apply In this case, consideration of continuity on a volumetric basis and use of Darcy's law lead to the following groundwater flow equation [3].

$$\frac{\partial \theta}{\partial t} = \nabla.(K \nabla h) \tag{1}$$

The left hand side of equation (1) is the rate of change of groundwater volume.

The hydraulic conductivity is defined as:

$$K = \frac{gk}{v} \tag{2}$$

Equation (1) can be rewritten as:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial h}{\partial z}\right) \tag{3}$$

which is a partial differential equation that cannot be solved directly due to two
unknown variables h and θ. If boundary conditions are prescribed, a unique solution can be obtained through numerical manipulation. The boundary conditions will usually be of two types:

- Potential conditions where the hydraulic potential is fixed
- Flux conditions where the groundwater flux perpendicular to the boundary is fixed, positive when entering flow domain.

In the zone of saturation, pressure potential are positive [3] and the groundwater volume θ, equals the porosity, n, where:

$$n = \frac{volume \;\; of \;\; voids}{total \;\; volume} \tag{4}$$

$$Hence \frac{\partial \theta}{\partial t} = \frac{\partial n}{\partial t} \tag{5}$$

in the zone of saturation.

Since porosity can change with water pressure, p,

$$\frac{\partial \theta}{\partial t} = C(1-n)\frac{\partial p}{\partial t} = pgC(1-n)\frac{\partial h}{\partial t} = s\frac{\partial h}{\partial t} \tag{6}$$

where C is the compressibility of the porous medium

and $\qquad$ $s = pgC(1-n) = \frac{\partial n}{\partial h}$ $\qquad$ (7)

Introducing equation (7) in equation (1)

$$s\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial h}{\partial z}\right) = \nabla(K\nabla h) \tag{8}$$

and hence for steady saturated groundwater flow in a homogeneous porous medium

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \nabla^2 h = 0 \tag{9}$$

which is the Laplace equation typical for potential problems.

FINITE ELEMENT FORMULATION

The code used in the current study is developed for two dimensional steady flow Thus equation (8) is reduced to:

$$\frac{\partial}{\partial x}\left(K\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial h}{\partial y}\right) = 0 \tag{10}$$

The finite element formulation of equation (10) is treated in detail in reference [3] and yields the following finite element equation.

$$\sum_{n=1}^{N} C_{mn}h_n = Q_m; m = 1 \cdot N \tag{11}$$

where $$C_{mn} = \int_D K\left(\frac{\partial B_m}{\partial x}.\frac{\partial B_n}{\partial x} + \frac{\partial B_m}{\partial y}.\frac{\partial B_n}{\partial y}\right)dxdy$$

$$Q_m = \int_D q_m B_n d_s$$

and $B_n(x,y)$ is the unification of all element interpolation functions that relate to node n. They are called basis functions. All basis functions are known.

In matrix form, equation (11) can be written as:

$$[C]\{h\} = \{Q\}$$

which is a set of algebraic equations.

The two dimensional steady state condition discussed above has been programmed for computer analysis of groundwater flow problems. The package, JALAM, was developed at the Laboratory of Hydrology Free University of Brussels. A description of the program is available in reference[4].

APPLICATION TO THE EWASO NYIRO BASIN

The finite element package, JALAM, was used in the prediction of groundwater potentials in the Ewaso Nyiro basin which is located in Central Kenya. The study area is shown in Figure 1.

From the available geological maps and rainfall data for the area, approximate zoning for values of transmissivity and rainfall rechage was done [5]. The zones and the values for transmissivity and rainfall rechage are shown in Figure 2 and 3 respectively.

The study area was subdivided into triangular finite elements as shown in Figure 4. The elements were carefully located taking into account factors such as:

(i) uniform transmissivity and rainfall recharge as much as possible over the element

(ii) location of some nodes on surface waters in order to prescribe the potentials for such nodes.

(iii) manipulation of nodes along the boundary of the study area in order to approximate the boundary as closely as possible.

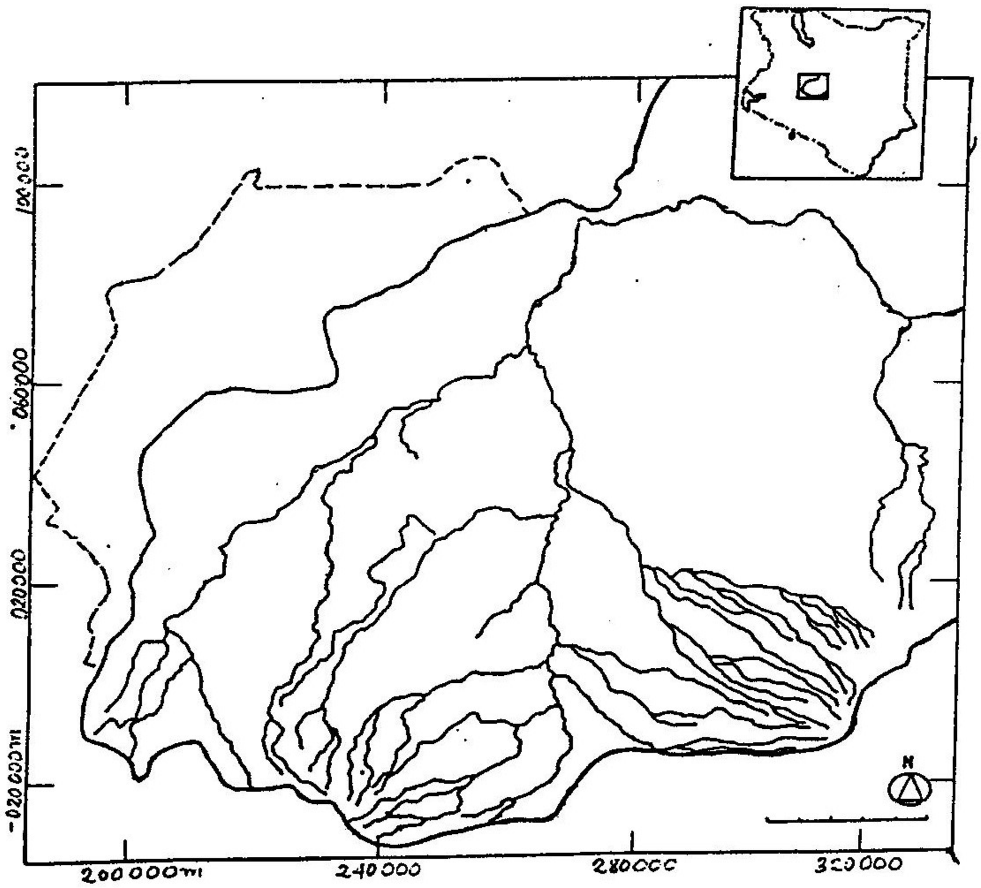

Fig. 1: Study area - the Ewaso Nyiro Basin in Central Kenya

The coordinates of all the nodes were obtained from the detailed geographical map of the study area. The same map was used to assign the fixed potentials for the nodes on the surface waters based on the contours. For all the nodes not located on surface waters, the flow was fixed as zero.

Using the basic data with respect to transmissivities and rainfall rechage from Figures 2 and 3 together with the information for the elements from Figure 4, all the necessary input data for program JALAM was prepared and analysis carried out.

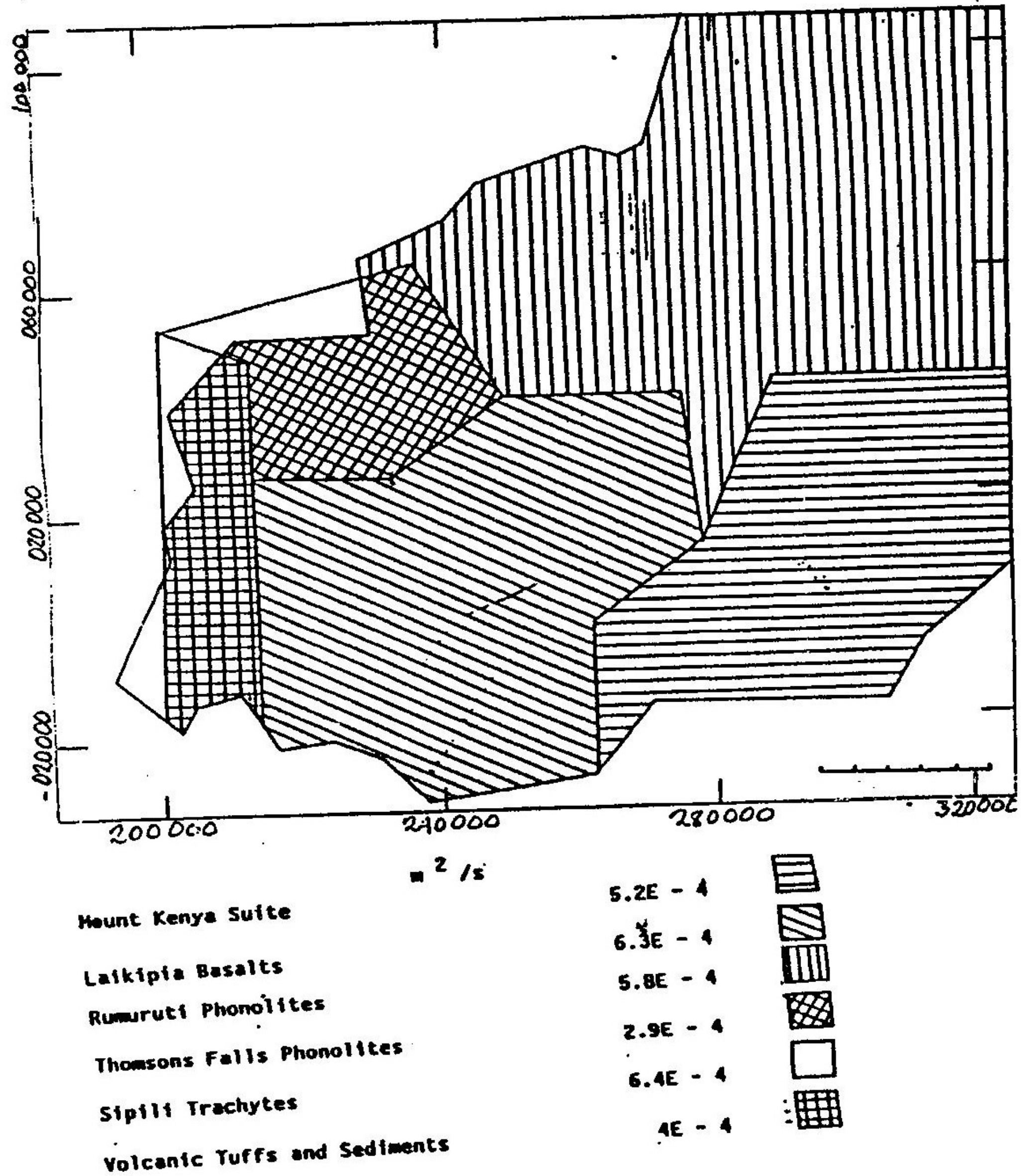

Fig.2: Profile of transmissivities

RESULTS AND DISCUSSIONS

Analysis with JALAM yielded the potential profiles shown in Figure 5. From the topology of the study area (see Figure 1), the profiles obtained in Figure 5 are in the expected flow directions of the basin.

Another researcher [5] has used the finite difference approach to calculate the groundwater potentials for the study area. His results are shown in Figure 6. It can be observed that the profiles obtained by the finite element

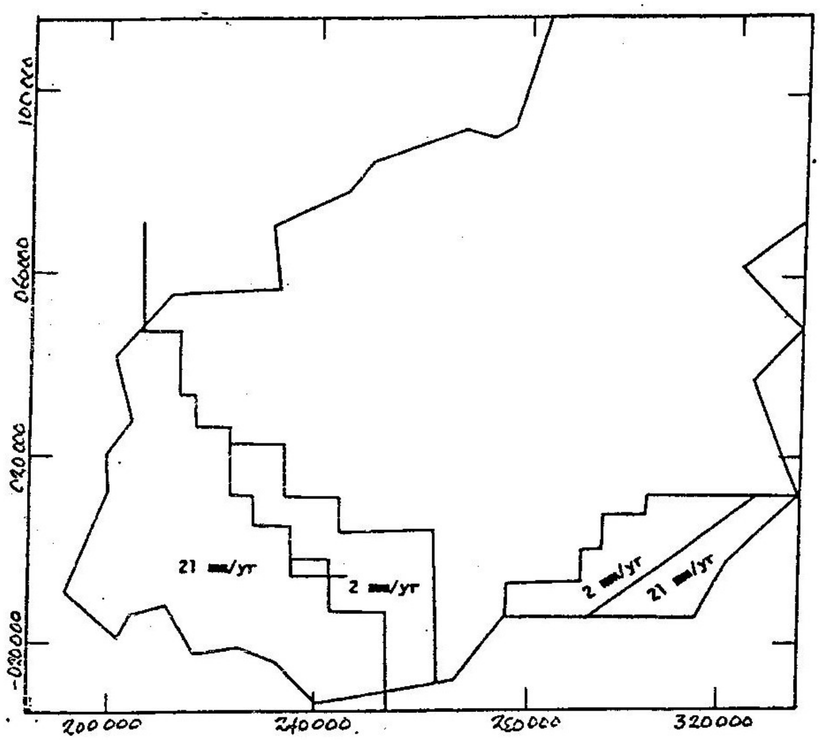

Fig 3: Distribution of rainfall recharge

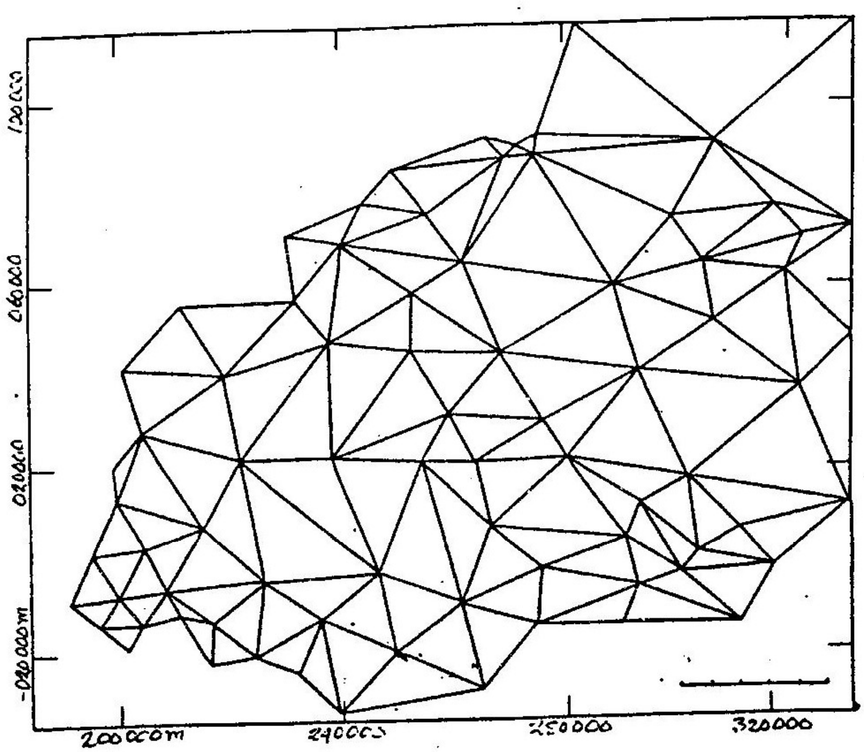

Fig.4: Finite element mesh

method are more consistent with the expected results than those of Figure 6 especially in the centre of the basin.

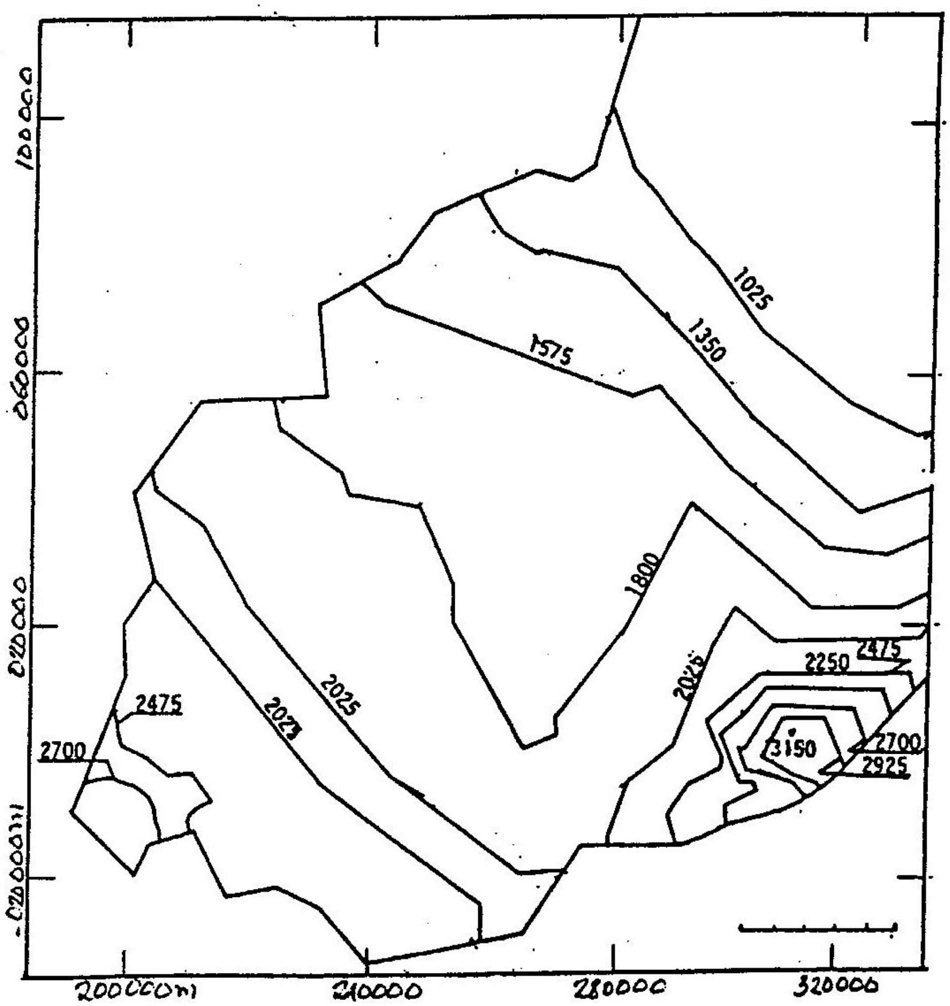

Fig. 5: Profiles of hydraulic potentials

The study area has some existing boreholes for which the location and depth to groundwater are known. The boreholes were drilled by the Water Resources Assessment Programme (WRAP), a department within the Ministry of Land Reclamation, Regional and Water Development of the Republic of Kenya from whom the data on location and depth to groundwater was obtained. Using a topographic map of the study area the boreholes were located since the coordinates are known. The elevation for each of the boreholes was calculated through interpolation of the contours. The groundwater potential, designated as the actual potential, was then deduced from the known depth. The positions of these boreholes were also located on the figure of the predicted potential profiles shown in Figure 5. Through interpolation again, the predicted groundwater potentials for the boreholes were obtained. The actual and interpolated potentials are compared on Table 1.

Table 1: Comparison of actual and predicted potentials

Borehole No.	Grid X (m)	Grid Y (m)	Ground level (m)	MWSL (m)	AP (m)	PP (m)	AP - PP (m)
2463	258861	082952	1779	99	1680	1366	314
7915	275565	090320	1410	63	1347	1129	218
1832	229161	057159	1908	227	1681	1738	-57
2280	232869	057158	1880	98	1782	1726	56
2400	242152	071899	1865	143	1722	1575	147
3200	240300	062686	1865	122	1743	1650	93
3201	249575	058988	1830	120	1710	1650	60
3413	249582	077426	1810	79	1731	1470	261
2259	255146	068210	1760	110	1650	1551	99
2750	262576	075573	1756	133	1623	1425	198
3430	275552	058979	1710	152	1558	1575	-17
3434	273703	055297	1700	45	1655	1595	60
3773	275556	070038	1530	111	1419	1442	123
3832	277429	099532	1210	17	1193	1025	168

MWSL - Mean Water Struck Level; **AP** - Actual Potential; **PP** - Predicted Potential

In general, the predicted potentials are less than the actual potentials and hence water will be struck sooner than the predicted depth. Where this is not the case, for instance, borehole numbers 1832, 3430 and 3773, the difference is small and conceivable within the error of prediction; the map used for determining the borehole elevations was published in 1988, has contour intervals of 60m and is to a scale of 1.250000.

The largest differences between the actual and predicted potentials were for the boreholes in the north of the catchment area. Inspection of the area showed that boreholes affected are located close to the divide of the catchment area. This could be due to differences in the orientation of the subsurface and surface flows in the specific location.

It should be noted that the model used in the analysis assumes saturated conditions. This could explain why the predicted depths are generally lager than the actual ones. During the drilling, groundwater can be encountered under unsaturated conditions.

The observed variance between the predicted and measured potentials for the study area could be further, due to the following:

(i) practically all the input data such as element coordinates, transmissivities and rainfall recharge, have been obtained by interpolation from maps of the study area. The coordinates and fixed potentials were obtained from a topographic map of the study area while the transmissivities and rainfall recharge were interpolated from limited measurements in the study area. These methods of data preparation reduce the accuracy of the model but were the only ones possible for the current study.

(ii) The zoning for the description of the transmissivities and rainfall recharge for the study area is only approximate since, as expected, actual boundaries are far more complex. In the case of rainfall recharge, seasonal changes will significantly affect measured potentials and hence variance with predicted values.

Considering the methods of data collection discussed above, a detailed error analysis is not reasonable at this stage of the research. It can be expected that actual measurement of borehole levels in the field and evaluation of transmissivities and rainfall recharge based on larger amounts of data will improve the results further. This process will also facilitate a fair error analysis.

However, the results obtained show that the finite element technique has a great potential in groundwater studies. The consistency of the hydraulic potentials with expected trends is encouraging. Its superiority over the finite difference technique is also demonstrated considering the results presented on Figure 5 and 6. This is besides its flexibility in handling complex domains as shown by the mesh in Figure 4. Numerical handling of the boundaries using the finite element method is also known to be easy unlike the case when using the finite difference technique.

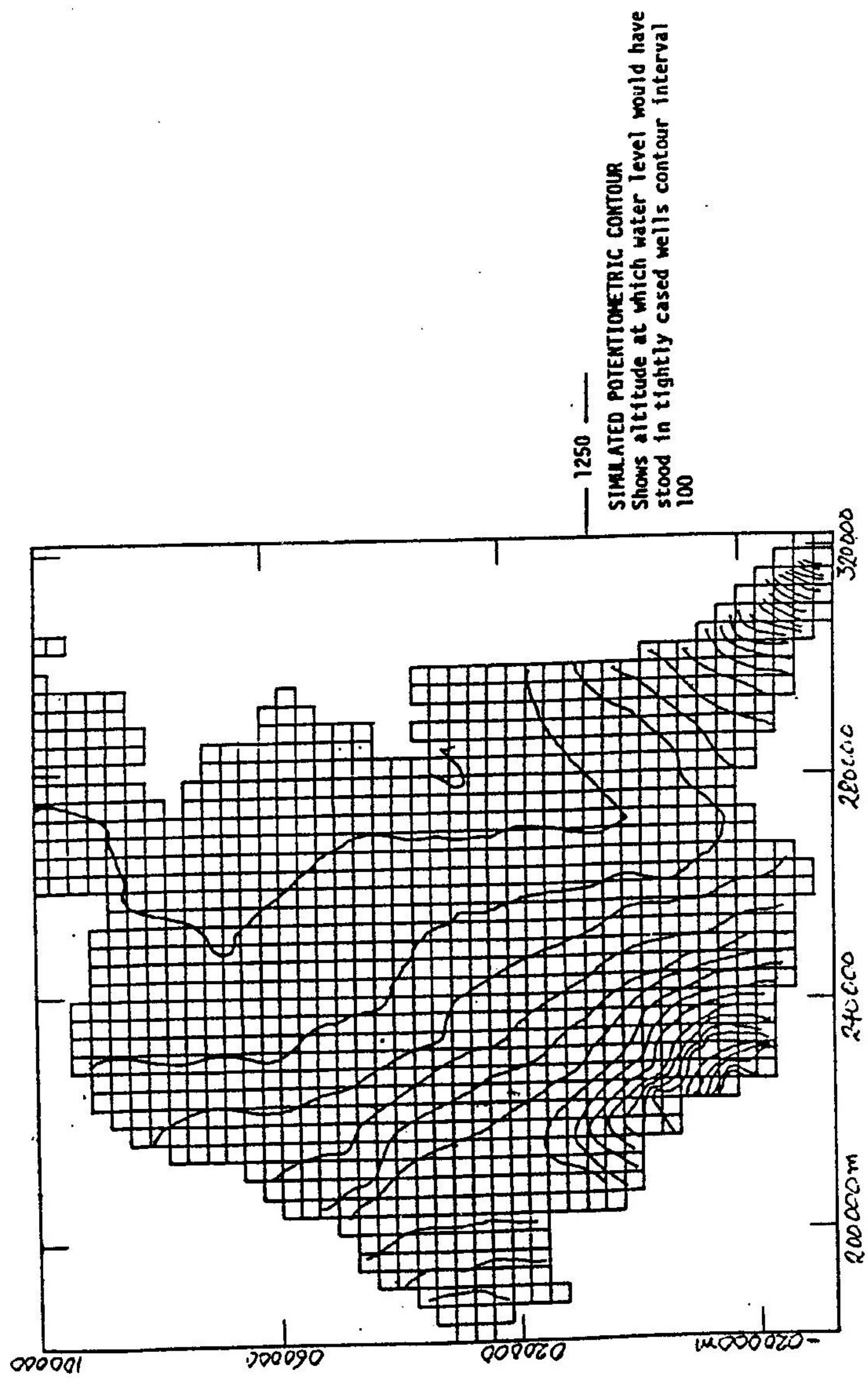

Fig. 6: Potential profiles using finite differences

The quality of the results also indicates that the concept of Reference Elemental Volume in the development of the model is reasonable and overcomes the complexities that would arise if a macroscopic approach were not employed.

CONCLUSIONS

(i) Based on the results in Figure 5, the finite element method can be used to plot the profiles for groundwater potentials.

(ii) As shown in Table 1, the finite element method has a great potential in the prediction of groundwater potentials and hence can be a useful tool in the location of boreholes in a given basis.

NOMENCLATURE

g = the acceleration due to gravity
h = is the groundwater potential
k = the permeability of the acquifer (proportionality constant)
K = the hydraulic conductivity
q_n = the flux in node n.
s = is the specific storage coefficient
v = the kinematic viscosity of the fluid.
θ = is the volume of water per Reference Elemental Volume (REV) on a macroscopic scale.

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