THEORETICAL PERFORMANCE EVALUATION OF AN INTEGRATED PACKET VOICE AND DATA CONCENTRATOR WITH OFFERED TRAFFIC

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ABSTRACT

An analytical model of an integrated packet voice and data concentrator which explicitly considers the traffic offered is presented. Assuming voice packets have pre-emptive priority over data packets performance measures, including delay distributions, are derived for both voice and data packets.

INTRODUCTION

Any terminal in an integrated packet voice and data system must incorporate a packet voice concentrator (PVC) (fig.1). In the packet voice concentrator speech from several sources is digitized at a uniform rate and encoded by an analog-to-digital (A/D) encoder and then organized into constant length packets by the packetizer. The speech activity detector (SAD) ensures that only packets containing active speech samples are actually transmitted.

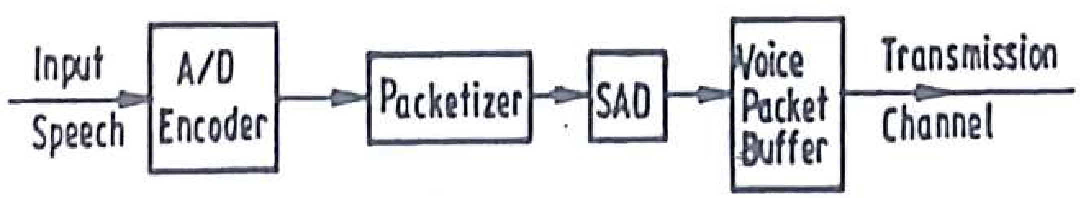

Fig. 1: A Packet voice concentrator

Fig.2, is a diagram of an integrated packet voice and data terminal as it might be incorporated in a switched telephone network. The packet voice concentrator accepts speech from N trunks and the voice packets generated by the concentrator are integrated with data packets and transmitted over a single wideband channel.

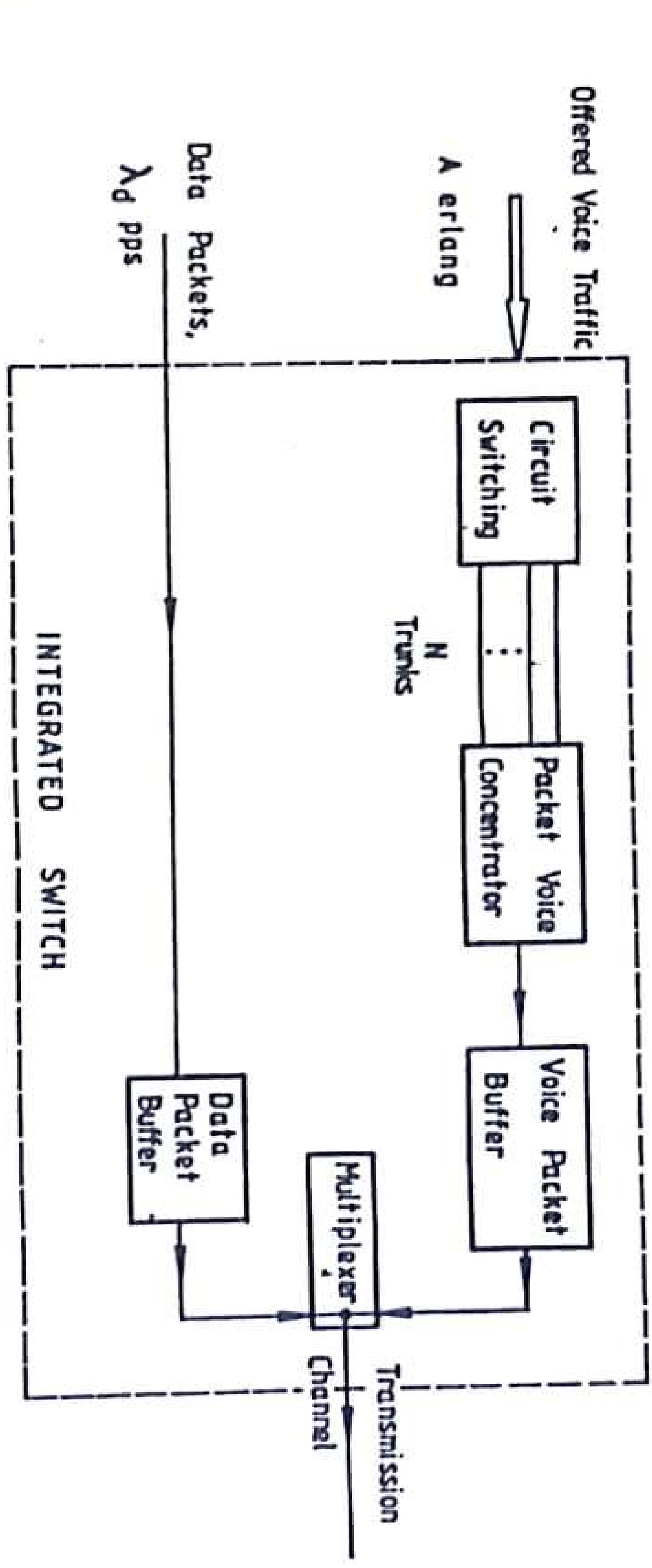

Fig.2. An Integrated Packet Voice and Data Multiplexer

We assume an Erlangian model for circuit-switching in which the N trunks represent a trunk group engineered to carryA erlangs of traffic. The circuit switching device connects an "off-hook" speech source to a trunk and maintains the connection until a source returns to the "on-hook" condition.

The buffer allows the N trunks and the data packets to share a single wideband transmission channel. Concentration is achieved since the total bit rate of the transmission channel available for transmission of voice packets is less than the total input bit rate of the N trunks when they are assumed to be all busy simultaneously.

The number of trunks N is chosen to meet a particular blocking probability (grade-of-service). The probability that i trunks are off-hook is given by the Erlang B-formula [1]:

$$\pi_i = \frac{A^i / i!}{\sum_{k=o}^{N} A^k / k!}, i = 0,1,\ldots,N \tag{1}$$

where A is the traffic intensity and is defined as $A = \lambda/\mu$ where λ is the average call arrival rate and μ^{-1} is the mean call holding time. The blocking probability, P_B, is given by $P_B = \pi_N$.

We adopt a model for a single speaker (call) which was originally proposed by Weinstein and Hofstetter [2] . In this model, each speech source is modelled as being either active (in "talkspurt") or silent. The holding times in the talkspurt and silent states are assumed to be exponentially distributed with means β^{-1} and α^{-1} respectively. Extensions of our model to handle the three state single speaker model are straightforward and given elsewhere [3] and in the appendix.

THE FLUID APPROXIMATION

We assume that each speaker in a talkspurt generates constant length packets at the rate of R packets per second (pps). Each voice packet thus has a length of l/R seconds and, without loss of generality, we shall assume that the data packet length is also l/R seconds. We shall further assume that the data packet arrival rate is Poisson at a rateλ_dR pps/ Note that when $\lambda_d = 0$, our system reduces to a pure packet voice communication system

[3,7].

We model the integrated concentrator buffer as having two distinct queues, one for voice packets and the other for data packets. Voice and data packets are transmitted over the wideband channel in a first-come-first-served (FCFS) discipline with voice packets having pre-emptive priority over data packets.

The wideband channel is assumed to have a capacity of CR pps. To be fair to data sources, we assume that a capacity C_dR pps of the transmission channel is reserved for the exclusive use of data packets. The remaining capacity $C_vR = (C-C_d)R$ pps is shared between voice and data packets with voice packets having pre-emptive priority.

Let x_V be the voice packet queue length, x_d be the data packet queue length and $P_{i,j}(x_v)$(respectively $P_{i,j}(x_d)$) be the equilibrium probability distribution function corresponding to i of the N trunks being off-hook, j of the off-hook trunks being in talkspurt and not more than x_v (respectively x_d) voice (respectively data) packets being in the voice (respectively data) queue.

We let $\pi_{i,j}$ represent the ergodic probability that i calls are off-hook and that j of the off-hook calls are in talkspurt. We can then show that

$$\pi_{i,j} = \frac{\frac{A^i}{i!}\binom{i}{j}\left(\frac{\alpha}{\beta}\right)^j}{\{1+\frac{\alpha}{\beta}\}^i\{\sum_{k=o}^{N}\frac{A^k}{k!}\}} \tag{2}$$

Using a result due to Lindley [4] and the expression for ξ given in eqn. (4) below we can show that the stability condition for the voice packet queue is given by

$$\sum_{i=1}^{N}\sum_{j=1}^{i} j\pi_{i,j} < C_v \tag{3}$$

Substituting for $\pi_{i,j}$ from eqn (2) above and simplifying shows that the stability condition may be rewritten as

$$\frac{\alpha A(1-P_B)}{C_v(\alpha+\beta)} < 1 \tag{3}$$

We shall assume that the stability condition is satisfied by the voice packet queue. We also assume that:

(i) all voice conversations begin with a talkspurt
(ii) calls may end even when in silence, but whenever a call ends and a talkspurt is present, that talkspurt must also end.

The two assumptions given immediately above are required to simplify the mathematical equations and do not severely compromise the accuracy of these types of models [5].

Given the assumptions above and using standard manipulation treating x_v as a continuous variable yields the following set of Kolmogorov forward equations

$$r_j \frac{d}{dx_v} p_{i,j}(x_x) = \lambda P_{i-1,j-1}(x_v) + (i+1-j)\alpha P_{i,j-1}(x_v)$$

$$+(i+1)\mu P_{i+1,j+}(x_v) + (j+1)\beta P_{i,j+1}(X_v)$$

$$-\{\lambda[\min(1, N-i)] + (i-j)\alpha + i\mu + j\beta\} P_{i,j}(X_v)$$

$$+(i+1)\mu\delta_{o,j} P_{i+1,j}(x_v) \tag{4}$$

where $i = 0, 1, 2, ..., N$
$j = 0, 1, ..., i$
$r_j = R(j - c_v)$
$\delta_{i,j}$ is the Kroenecker delta

and $P_{i,j}(x_v) = 0$ for $i < 0$, $j < 0$, $j > i$ or $i > N$.
Putting the set of equations in (4) into matrix notation yields

$$\underline{D}\frac{d}{dx_v}\underline{P}(x_v) = \underline{M}\,\underline{P}(x_v) \qquad (5)$$

where $\underline{p}(x_v)$ is a column vector of distribution functions whose elements are the $P_{i,j}(x_v)$'s, $\underline{D}$ is a diagonal matrix whose elements are the r_j's and $\underline{M}$ is an infinitesimal generator for the call arrival and departure process and the activity of the off-hook calls going into and out of talkspurt. To ensure uniformity in equations we arrange the elements of the vector$\underline{p}(x_v)$ so that the indices (i,j) are in lexicographic order

The solution of equation (5) for the voice subsystem which is represented by equation (4) can be shown to be

$$P_{i,j}(x_v) = \pi i,j + \sum_{n=0}^{N-I_0} k_n \{\Phi_n\}_{(i,j)} e^{s_n x_v} \qquad (6)$$

for i=0,1,...,N; j=0,1,...,i and where s_n is the n^{th} negative eigenvalue of $\underline{D}^{-1}\underline{M}$, Φ_n is a right eigenvector of $\underline{D}^{-1}\underline{M}$ corresponding to s_n, $\{\phi_n\}_{(1,j)}$ is an element of Φ_n corresponding to $p_{i,j}(x_v)$, the k_n are normalizing constants and $I_0 = [c_v] + 1$ where [.] represents the integer part of its argument.

Voice packet queue contents grow monotonically whenever the number of trunks in talkspurt is greater than $I_0 - 1$: In that case, the probability that a voice packet queue is empty is zero. Using this fact in equation (6) yields:

$$\pi_{i,j} = -\sum_{n=0}^{N-I_0} k_n \{\Phi_n\}_{(i,j)} \qquad (7)$$

for $i = I_0, I_0+1, \ldots, N$; $j = I_0, I_0+1, \ldots, i$.

The set of simultaneous equations given in (7) is sufficient to solve for the normalizing constants k_n.

For data packets, the set of forward equations is also given by equation (4) but with x_d substituted for x_v and $r^*_{i,j}$ given by equation (8) below being substituted for r_j:

$$r^*_{i,}j = R\{\lambda_d - \max(C_{d,}C - j)\}\tau_{i,j} + R(\lambda_d - C_d)(1 - \tau_{i,j}) \qquad (8)$$

where $\tau_{i,j}$ is the conditional probability that $x_v = 0$ given that i calls are off-hook and j of the off-hook calls are in talkspurt. The conditional probabilities $\tau_{i,j}$ are easily determined from eqn. (6) and are given by:

$$\tau_{i,j} = 1 + \frac{1}{\pi_{i,j}} \sum_{n=0}^{N-I_0} k_n \{\Phi_n\}_{(i,j)} \tag{9}$$

Following Lindley [4] we find that the data packet queue will be stable provided:

$$\sum_{i=0}^{N} \sum_{j=0}^{i} r^{*}_{i,j} \pi_{i,j} < 0 \tag{10}$$

For the data packet subsystem, the solution to eqn.(5) with the elements of $\underline{D}$ obtained from eqn. (8) is given by an equation similar to eqn. (6). The number of negative eigenvalues of $\underline{D}^{-1}\underline{M}$ for the data packet subsystem may be shown to be obtainable from the condition $r^{*}_{i,j} > 0$. The number I'_0 corresponding to I_0 in eqn. (6) may be shown to be given by:

$$I'_o = \sup[c_v + \frac{1}{\tau_{i,j}}(c_d - \lambda_d)] \tag{11}$$

$i = 0,1, ..., N$
$j \varepsilon \underline{I}_u$
$j \leq i$

where $\underline{I}_u$ is the set of active voice calls j such that

$j = 0,1,2, ..., I_0 - 1$

sup(.) represents the "superior of"

[.] represents the smallest integer greater than its argument.

Equation (11) enables us to conclude that the number of negative eigenvalues, n_d, for the data packet subsystem is given by:

$$n_d = n - I'_o + 1 \tag{12}$$

The solution of equation (5) for the data subsystem is given by

$$P_{i,j}(x_d) = \pi_{i,j} + \sum_{n=0}^{n_d-1} k'_n \{\Phi'_n\}_{(i,j)} e^{s'_n x_d} \tag{13}$$

for $i = 0,1, ...,N$; $j=0,1, ...,i$ and where k'_n, ϕ'_n and s'_n are defined exactly like k_n, ϕ'_n and s_n were defined previously for the voice subsystem.

The data subsystem is designed with c_d <symbol$_d$ because extra capacity for data packet transmission is obtained by transmitting data during silent

periods in voice conversations. Thus, the data packet queue grows monotonically whenever the number of trunks in talkspurt is greater than or equal to I'_0. We, therefore, solve for the normalizing constants, k'_n, from

$$\pi_{i,j} = -\sum_{n=0}^{n_d-1} k_n \{\Phi_n\}_{(i,j)} \tag{14}$$

for $i = I'_0,\ I'_0 + 1, \ldots, N;\ \ j = I'_0,\ I'_0 + 1, \ldots, i$.

PERFORMANCE MEASURES

The voice packet delay distribution is given by

Pr {voice packet delay $\leq$ t sec} =

$$\sum_{i=0}^{N}\sum_{j=0}^{i} p_{i,j}(c_v Rt) \tag{15}$$

with $p_{i,j}(.)$ being obtained from eqn. (6).

For voice packets, another important performance measure is the fractional packet loss when the voice packet queue has finite capacity If the maximum size of the voice packet queue is set at ϕ_m packets, then the fractional packet loss, L, is given by

$$L = \frac{100}{\eta}\sum_{i=I_0}^{N}\sum_{j=I_0}^{i}(j - c_v)\{1 - p_{i,j}(\Phi_m)\}\% \tag{16}$$

with $p_{i,j}(.)$ being obtained from eqn. (6) and where

$$\eta = \sum_{i=j}^{N}\sum_{j=0}^{i} j\pi_{i,j} \tag{17}$$

is the mean number of active calls.

The argument leading to eqn. (8) enables us to show that the data packet delay distribution is given by

Pr {Data packet delay distribution $\leq$ t sec} =

$$\sum_{i=0}^{N}\sum_{j=0}^{i}\{p_{i,j}(\max[c_d,(c-j]Rt)]\tau_{i,j}+[p_{i,j}(c_dRT)](1-\tau_{i,j})\} \quad (18)$$

CONCLUSION

This paper has extended the use of the two state single speaker model to the analysis of packet voice and data systems using the three state single speaker model. Validation of the model via computed results is straightforward if one has necessary software to compute the eigenvalues and eigenvectors of relatively unstable matrices of very lage dimensions and this is left to the reader. Confidence in the theoretical model is derived from the fact that the model reduces to the two state single speaker model for which computational results are available and they compare very well to experimental results [3,7].

NOMENCLATURE

π_i	Probability that i trunks are off-hook
λ	Average call arrival rate
μ^{-1}	Mean call holding time
P_B	Blocking probability
α^{-1}	Mean silence length
β^{-1}	Mean talkspurt length
R^{-1}	Mean voice or data packet length
$\lambda_a R$	Data packet arrival rate
c	Wideband channel capacity
c_d	Channel capacity reserved for data packets
x_v	Voice packets in a queue
x_d	Data packets in a queue
$\pi_{i,j}$	Ergodic probability that i voice calls are off-hook and j of the off-hook calls are in talkspurt
$\delta_{i,j}$	Kronecker delta
$\underline{D}$	Diagonal matrix for the packet arrival process
$\underline{M}$	Infinitesional generator for the call arrival and departure process.
s_n	n^{th} negative eigenvalue of $\underline{D}^{-1}\underline{M}$ for the voice subsystem
s'_n	n^{th} negative eigenvalue of $\underline{D}^{-1}\underline{M}$ for the data subsystem

ϕ_n	Right eigenuector corresponding to s_n
ϕ'_n	Right eigenvector corresponding to s'_n
[.]	The integer part of its argument
I_n	$[c^v] + 1$ for the voice subsystem
I_u'	Refers to the data subsystem but corresponds to I_n for the voice subsystem
24.	Maximum size of the voice queue
L	Fractional packet loss
η	Mean number of active voice calls.

APPENDIX

EXTENSION OF THE MODEL TO THE THREE STATE SINGLE SPEAKER MODEL

So far in this paper we have modelled the activity of a single speaker (call) using a two state Markov model. If this model is used to represent the activity of each speaker in a population of N speakers then the number of active speakers in the population may be represented by an N state Markov chain.

Focus attention on the talkspurt interarrival times for an aggregate process representing the activity of a population of N speakers. Experimental results [2] have shown that when N>25, the talkspurt interarrival times for the aggregate process are exponentially distributed thus validating the two state single speaker model. The same experimental results have shown, however, that when $N < 10$, the talkspurt interarrival times were not exponentially distributed. This stems from the fact that the silence length distribution for a single speaker is not exponentially distributed [2]. To more accurately model the aggregate process of N speakers when $N < 10$, a three state single speaker model which yields non-exponentially distributed silence length distributions has to be used. The three state single speaker model is shown in the figure 3.

Short silences are more frequent than long silences [6,7]. Hence $_1$, β_1 >> $_2$, β_2. Making appropriate assumptions as was done in section 2 and defining $p_{i,j,k}(x_v)$ to be the equilibrium probability distribution function corresponding to i calls off-hook, j calls in talkspurt, k calls in short silence and not more than x_v packets in the voice packet queue yields the following set of Kolmogorov forward equations:

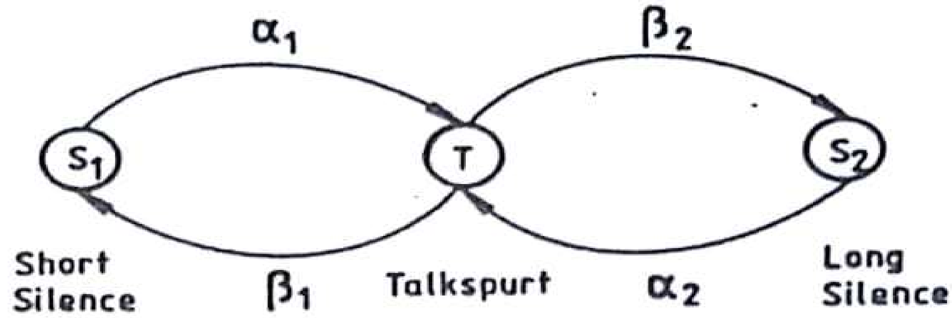

Fig. 3 The three state single speaker model

$$r_j \frac{d}{dx_v} p_{i,j,k}(x_v) = \lambda p_{i-1,j-1,k}(x_v) + (i+1-j-k)\alpha_2 p_{i,j-1,k}(x_v)$$
$$-\{\lambda[\min.1, N-i] + +(i-j-k)\alpha_2 + k\alpha_1 + i\mu + j(\beta_1 - \beta_2)\} p_{i,j,k}$$
$$+(k+1)\alpha_1 p_{i,j-1,k+1}(x_v) + (j+1)\beta_1 p_{i,j+1,k-1}(x_v)$$
$$+(j+1)\beta_2 p_{i,j+1,k}(x_v) + (i+1)\mu p_{i+1,j+1,k}(x_v)$$
$$+(i+1)\mu\delta_{o,j,k} p_{i+1,j,k}(x_v) \qquad \text{(A.1)}$$

where $i = 0,1, ..., N$
$j = 0,1, ..., i$
$k = 0.1, ..., i-j$
$r_j = R(j - c_v)$
$\delta i,j,k$ is the Kronecker delta
and $p_{i,j,k}(x_v) = 0$ for $i<0$, $j<0$, $k<0$, $j>i$, $k>i$, or $i>N$.

Using a result due to Lindley [4] we may show that the stability condition for the voice packet queue is given by

$$\sum_{i=1}^{N}\sum_{j=p}^{i}\sum_{k=0}^{i-j} j\pi_{i,j,k} < c_v \qquad \text{(A.2)}$$

where $\pi_{i,j,k}$ is the ergodic probability that i calls are off-hook, j calls are in talkspurt and k calls are in short silence. It can be shown that:

$$\pi_{i,j,k} = \frac{\frac{A^i}{i!}\binom{i}{j+k}\binom{j+k}{k}\left(\frac{\alpha_2}{\beta_2}\right)^{j+k}\left(\frac{\beta_1}{\alpha_1}\right)^k}{\{\sum_{i=0}^{N}\frac{A^1}{1!}\}\{\frac{\alpha_2}{\beta_2}\left(1+\frac{\beta_1}{\alpha_1}+\frac{\beta_2}{\alpha_2}\right)^i} \qquad (A.3)$$

Substituting eqn (A.3) into eqn (A.2) and simplifying yields the following as the equilibrium condition for the voice packet queue

$$\frac{A\{1-p_\beta\}}{c_v(1-\frac{\beta_1}{\alpha_1}+\frac{\beta_2}{\alpha_2})} < 1 \qquad (A.4)$$

Equations similar to eqn. (A.l) and a stability condition similar to eqn.(A.4) can be derived in a similar manner for the data packet queue. The derivations are left as an exercise to the reader

Note that when $\alpha_2 = 0$, $\pi_{i,0,0}$ reduces, as expected, to the Erlang - ß formula (eqn.1). Note further that when $ß_1 = 0$, eqn. A.3 reduces, as expected, to equation eqn.2.

Equation (A.l) can be solved in a manner parallel to the approach in section 2 of this paper. Note, however, that whereas previously (using the two state single speaker model) we had a state space with (N+l) (N+2) states, now, using the three state single speaker model we have a state space consisting of (N+l) (N+2) (N+3) states. Thus even for a relatively small system with N=10, the matrix M, which previously had dimensions of 66x66 now has dimensions of 286 x 286! Thus although the fluid approximation model may, in principle, be extended to handle the three state single speaker model, the resulting size of the state space may considerately reduce the utility of these models in the analysis of integrated packet voice and data systems with offered traffic.

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